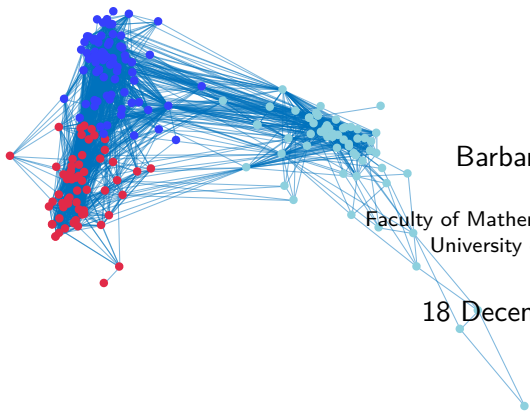


Evolutionary Dynamics on Evolving Graphs



Barbara Ikica

Faculty of Mathematics and Physics
University of Ljubljana

18 December 2019

Evolutionary Dynamics on Evolving Graphs

Motivation

Outline

Evolutionary Dynamics on Graphs

The Modified
Petford-Welsh
Algorithm

Evolving Graphs

Co-evolution of the
Multilayer News Flow

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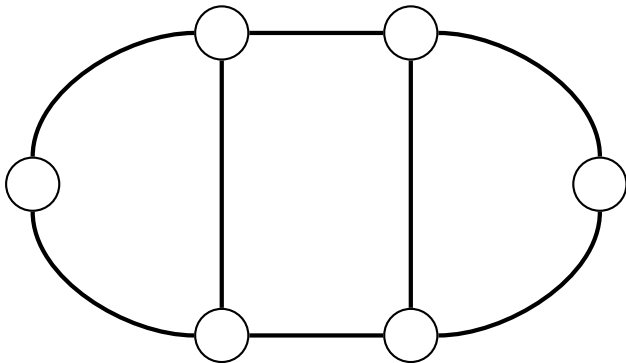
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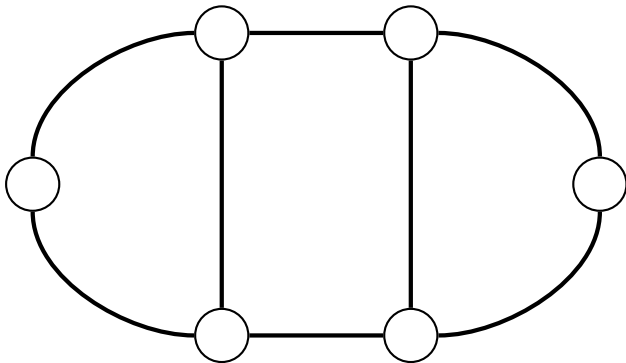
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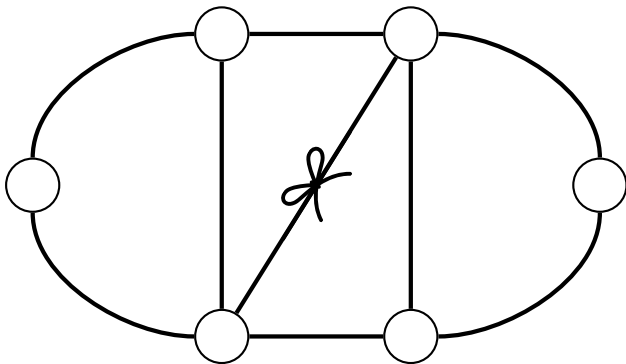
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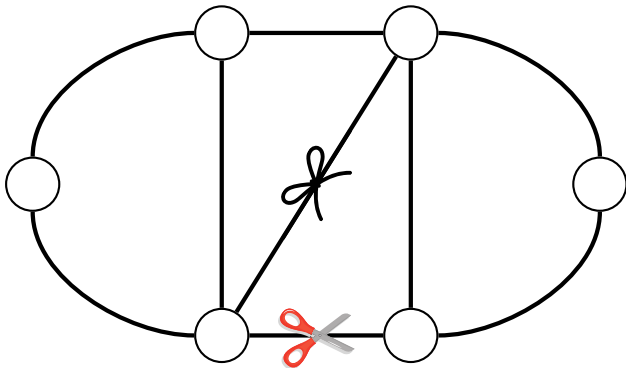
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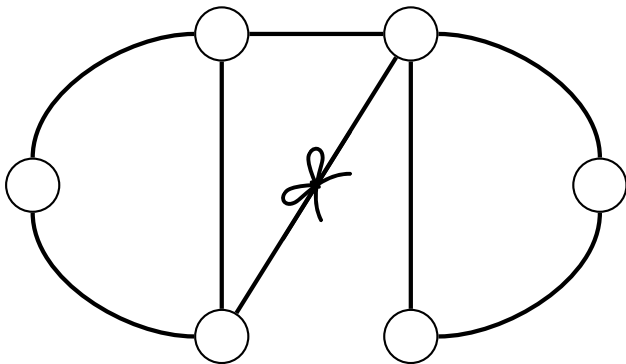
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Evolutionary Dynamics on Evolving Graphs

Players/agents $\mathcal{P} = \left\{ \text{♂}, \text{♀}, \text{♂}, \text{♀}, \text{♂}, \text{♀} \right\}$

Evolutionary Dynamics on Evolving Graphs

Players/agents $\mathcal{P} = \left\{ \text{👤}, \text{👩}, \text{👨}, \text{👧}, \text{👦}, \text{👷} \right\}$

Strategies/states $\mathcal{S}_i = \left\{ \text{📄}, \text{✂️}, \text{📱} \right\}$

Evolutionary Dynamics on Evolving Graphs

Players/agents $\mathcal{P} = \left\{ \text{cup}, \text{woman}, \text{man}, \text{woman}, \text{man}, \text{woman} \right\}$

Strategies/states $\mathcal{S}_i = \left\{ \text{stone}, \text{scissors}, \text{paper} \right\}$

Payoff $\pi_i : \left\{ \left(\text{cup}, \text{stone}, \text{woman}, \text{scissors}, \text{man}, \text{paper}, \text{woman}, \text{stone}, \text{man}, \text{scissors}, \text{woman}, \text{paper} \right) \right\} \rightarrow \text{€}$

Evolutionary Dynamics on Evolving Graphs

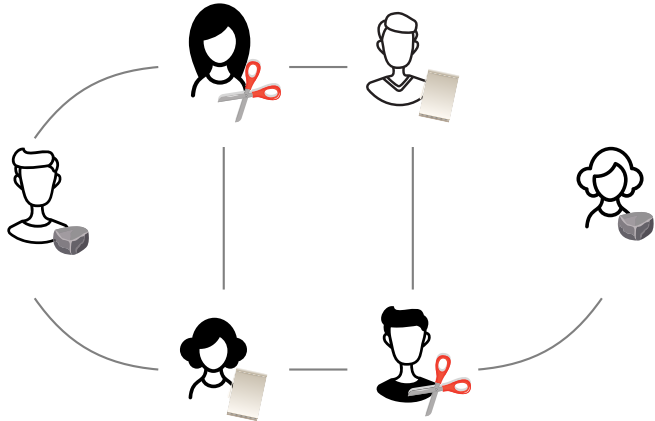
Players/agents $\mathcal{P} = \left\{ \text{cup}, \text{woman}, \text{man}, \text{woman}, \text{man}, \text{woman} \right\}$

Strategies/states $S_i = \left\{ \text{cube}, \text{scissors}, \text{card} \right\}$

Payoff $\pi_i : \left\{ \left(\text{cup}, \text{scissors}, \text{man}, \text{card}, \text{man}, \text{scissors}, \text{woman} \right) \right\} \rightarrow \text{€}$

Update rule $p_{i \rightarrow j} = \left(1 + e^{-\omega[\pi_i - \pi_j]} \right)^{-1}$ (Fermi rule)

Evolutionary Dynamics on Evolving Graphs



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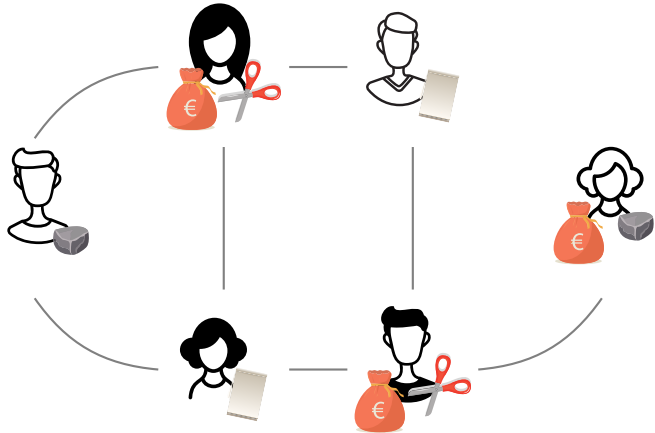
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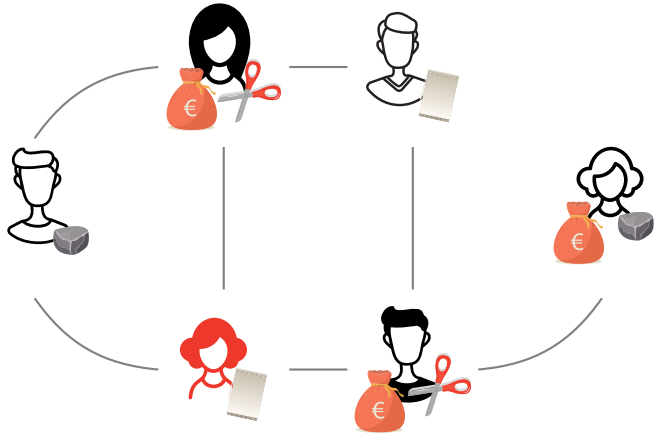
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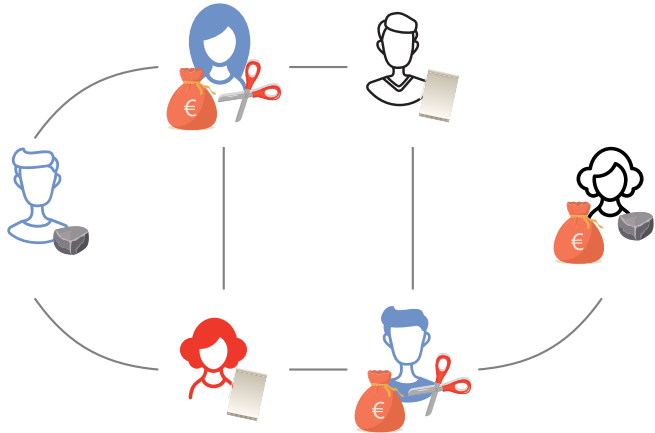
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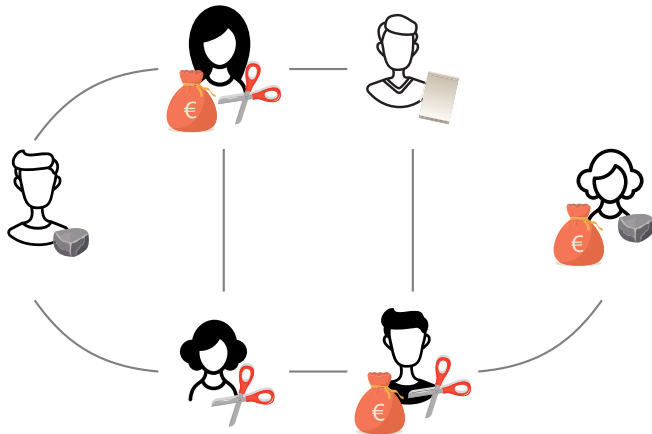
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Evolutionary Dynamics on Evolving Graphs



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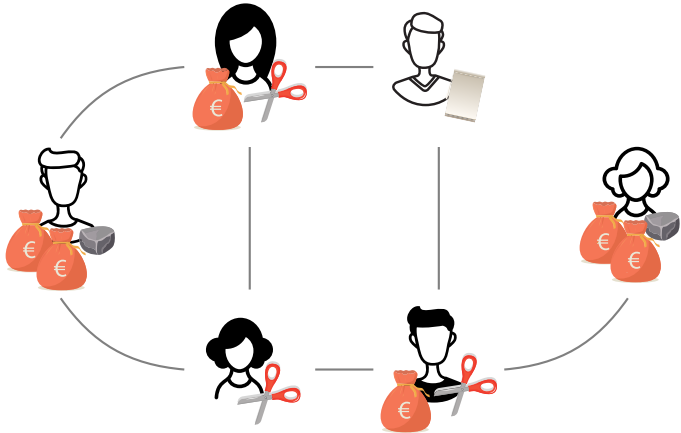
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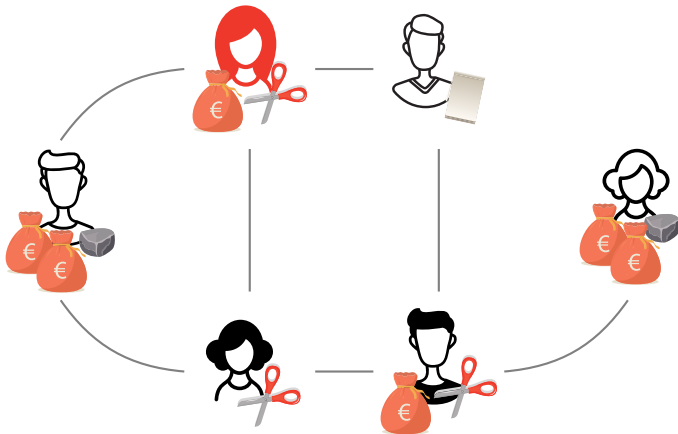
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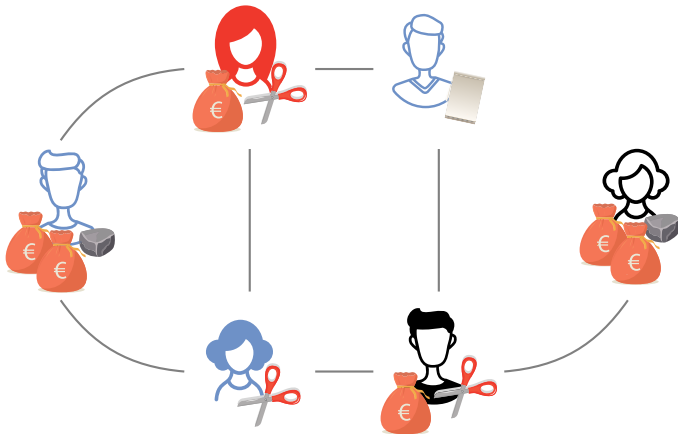
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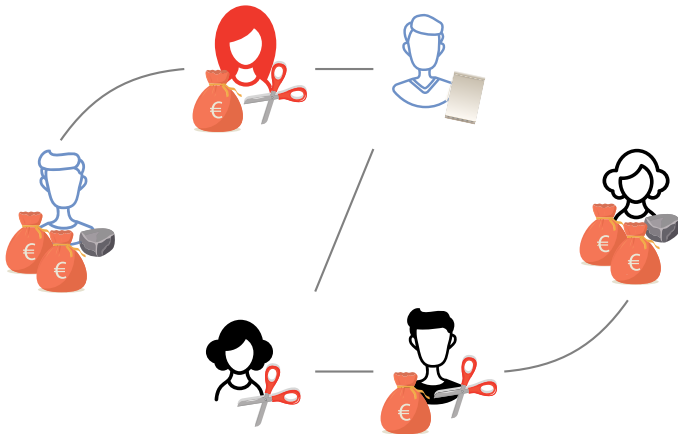
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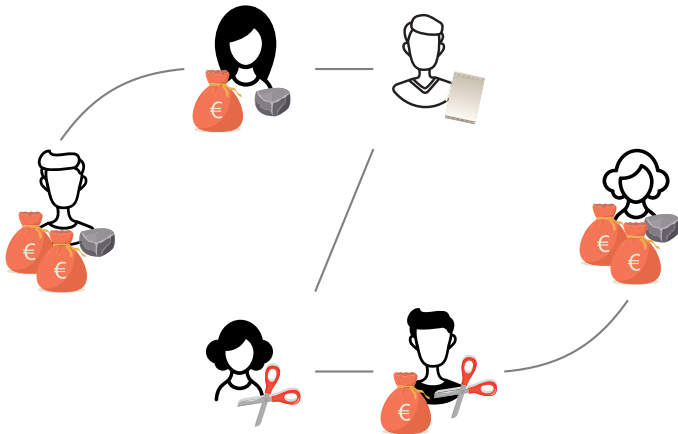
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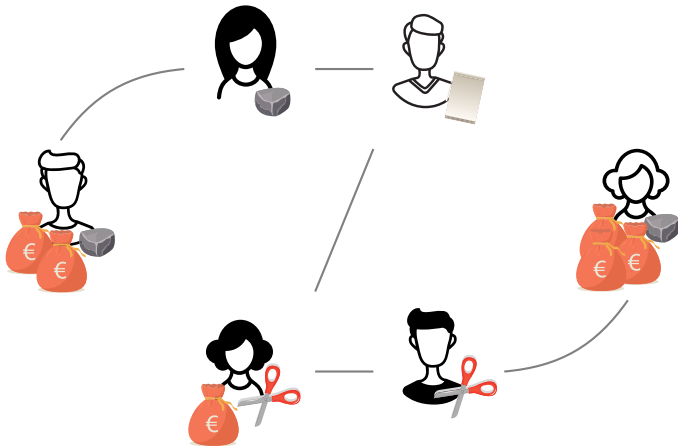
Evolutionary Dynamics on Evolving Graphs



Evolutionary Dynamics on Evolving Graphs



Evolutionary Dynamics on Evolving Graphs



Evolutionary Dynamics on *Evolving* Graphs

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Evolutionary Dynamics on Evolving Graphs

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$$t_D \gg t_G$$

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$$t_D \sim t_G$$

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$$t_D \ll t_G$$

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$$t_D \sim t_G$$



News providers



$$t_D \sim t_G$$

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News providers



News consumers

$$t_D \sim t_G$$

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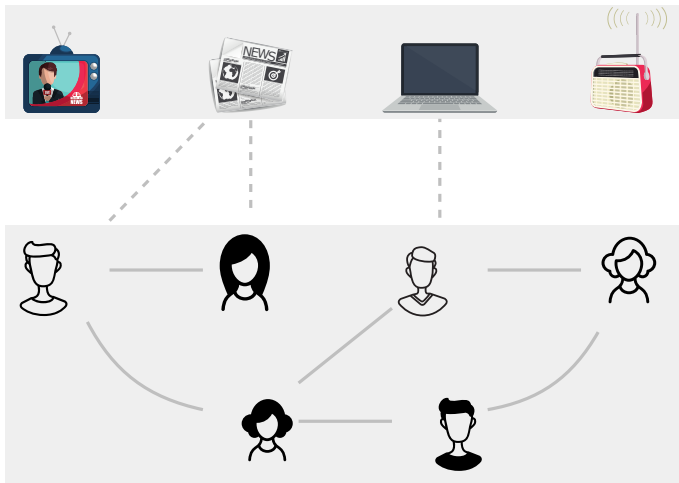
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$$t_D \sim t_G$$

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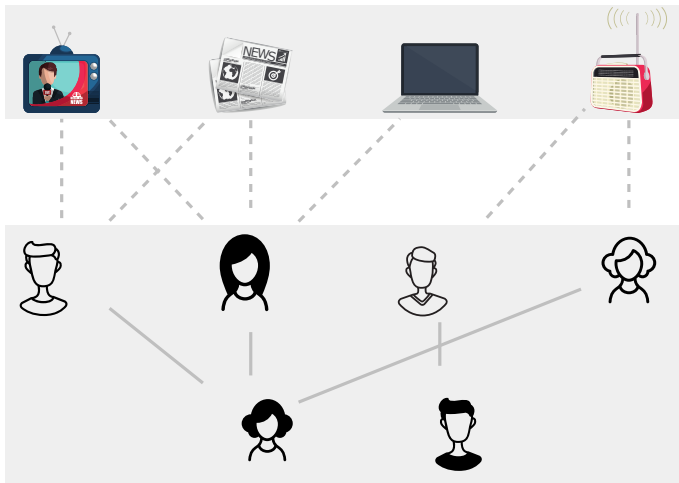
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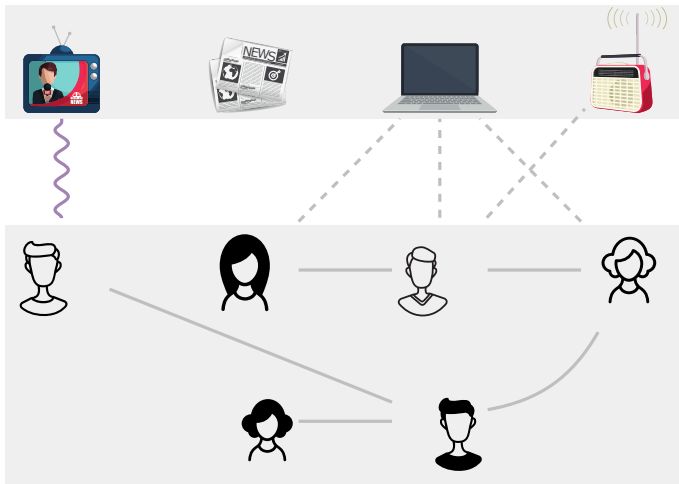
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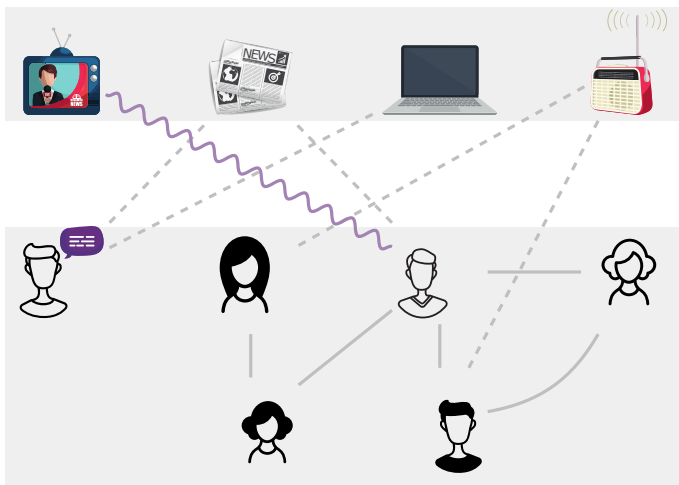
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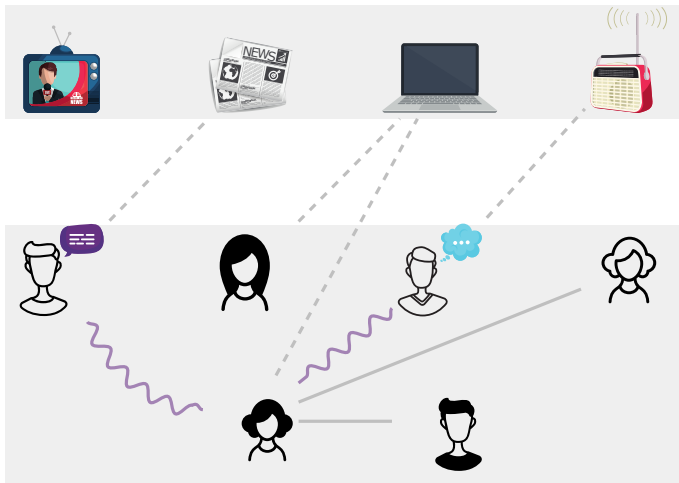
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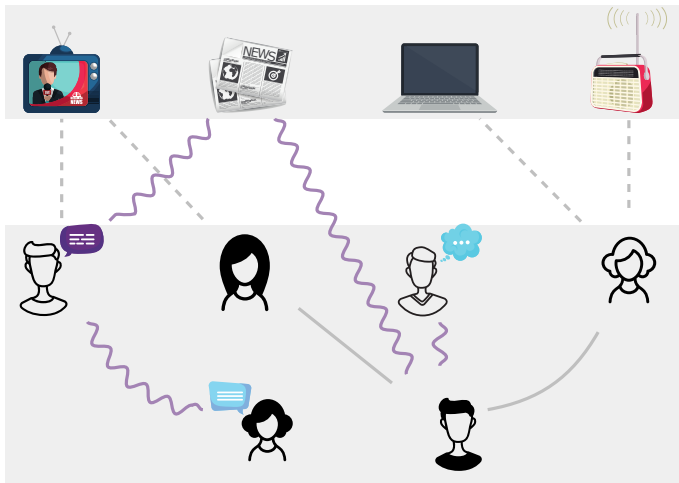
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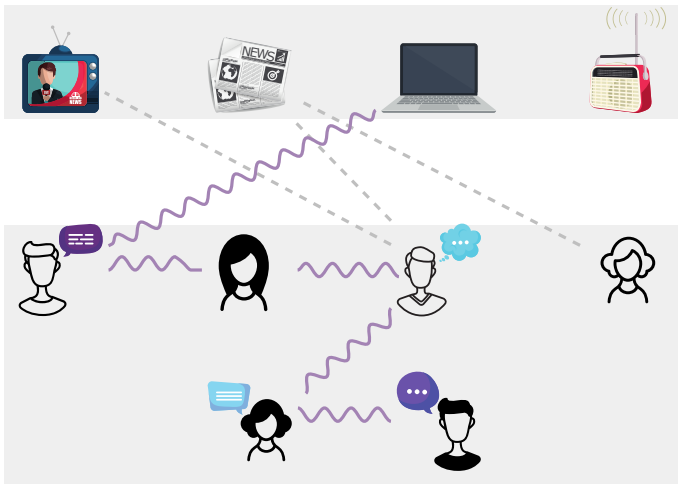
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Evolutionary Dynamics on Evolving Graphs

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t_D
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$$t_D \gg t_G$$

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I Evolutionary Dynamics on Graphs ($t_D \ll t_G$)

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- *The Modified Petford–Welsh Algorithm*

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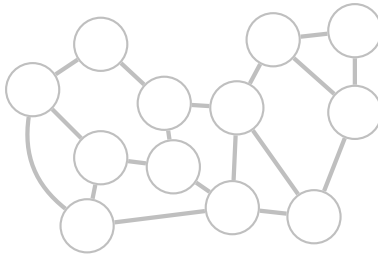
I Evolutionary Dynamics on Graphs ($t_D \ll t_G$)

- *The Modified Petford–Welsh Algorithm*

II Evolving Graphs ($t_D \sim t_G$)

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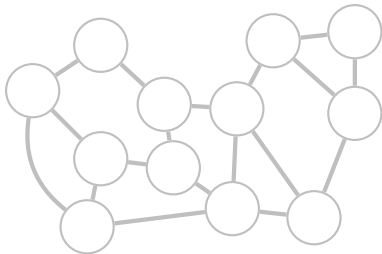
The Modified Petford–Welsh Algorithm



The Modified Petford–Welsh Algorithm

Petford–Welsh algorithm

Petford, A. D. & Welsh, D. J. A. A Randomised 3-Colouring Algorithm, *Discrete Math.* **74** (1989), 253–261.

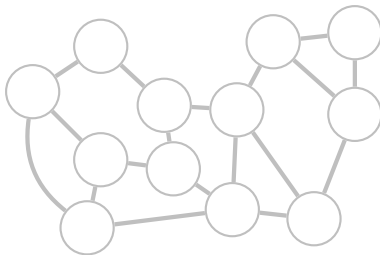


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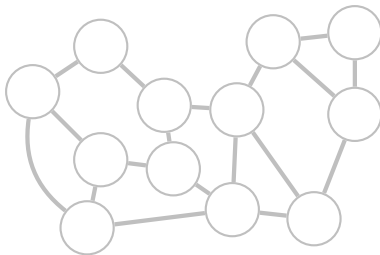


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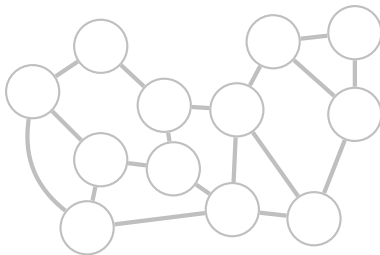
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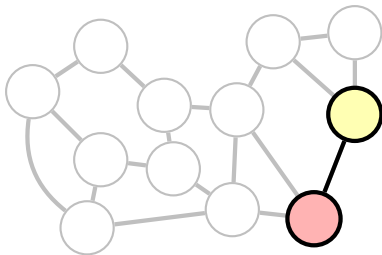
The Modified Petford–Welsh Algorithm

A **proper colouring** is an assignment of colours to the vertices of a graph so that no two adjacent vertices have the same colour, i.e., $c : V \rightarrow \{1, 2, \dots, k\}$ s.t. $c(i) \neq c(j)$ for all $ij \in E$.



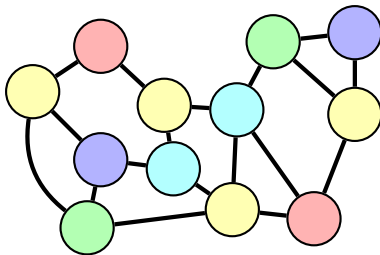
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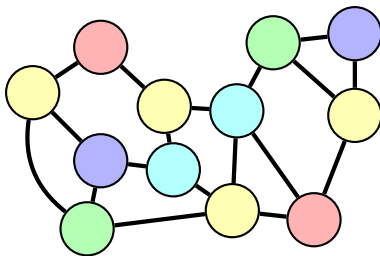
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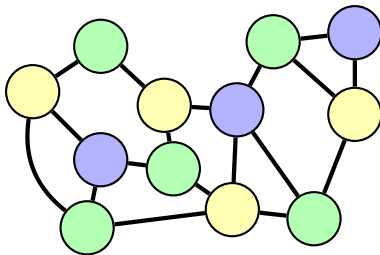
A graph that has a k -colouring is said to be k -colourable.



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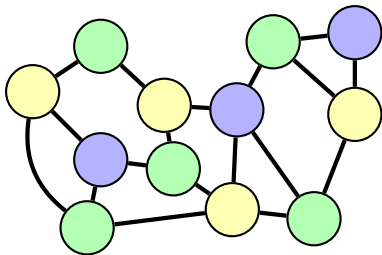


The Modified Petford–Welsh Algorithm

Petford–Welsh algorithm

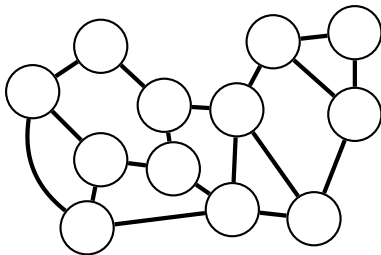
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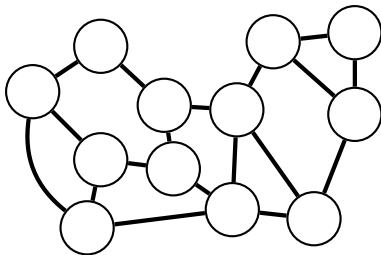
A randomised k -colouring algorithm



The Modified Petford–Welsh Algorithm

A randomised k -colouring algorithm

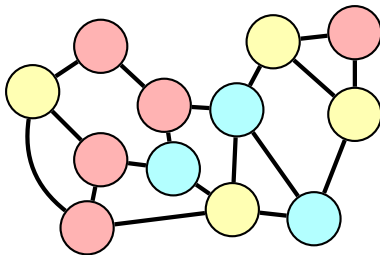
1. generate an initial k -colouring



The Modified Petford–Welsh Algorithm

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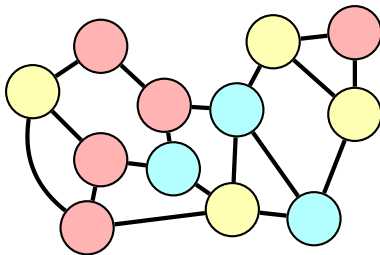
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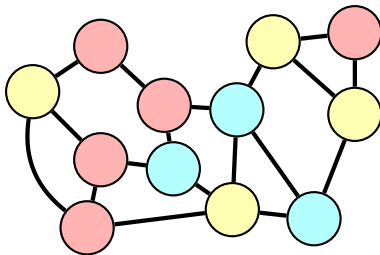
1. generate an initial k -colouring
2. **while** (there is a *bad vertex*) **and** (not too many steps) **repeat**



The Modified Petford–Welsh Algorithm

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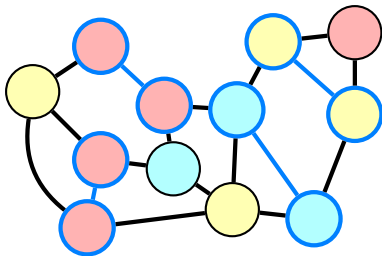
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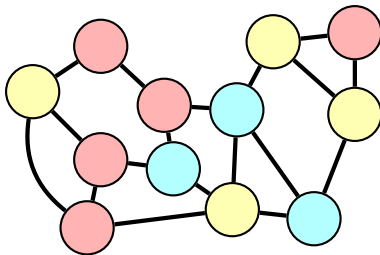
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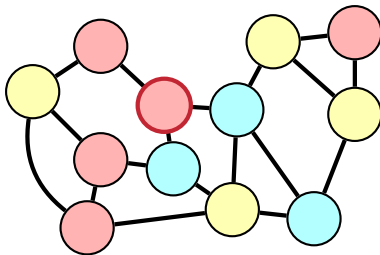
1. generate an initial k -colouring
2. **while** (there is a *bad vertex*) **and** (not too many steps) **repeat**
 - 2.1 choose a bad vertex v uniformly at random



The Modified Petford–Welsh Algorithm

A randomised k -colouring algorithm

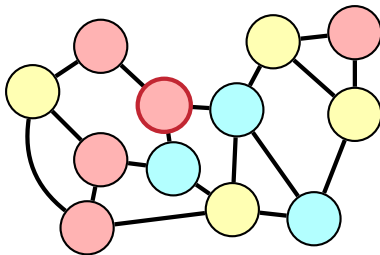
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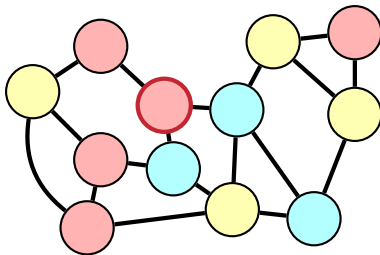
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$$\mathcal{N}(\text{red}, r) = 1$$

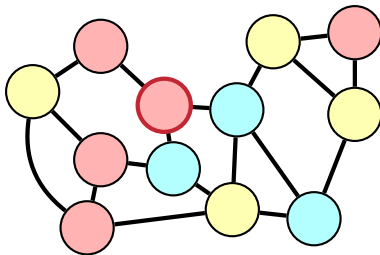
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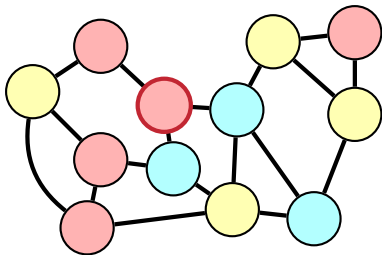
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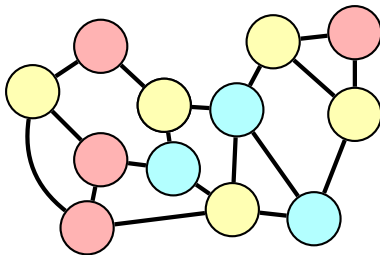
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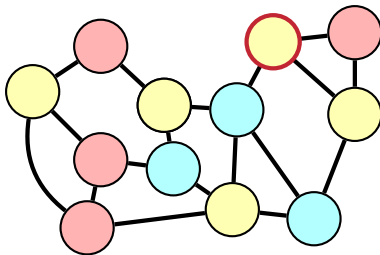
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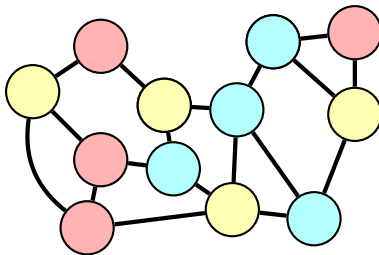
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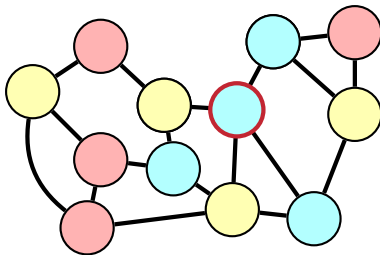
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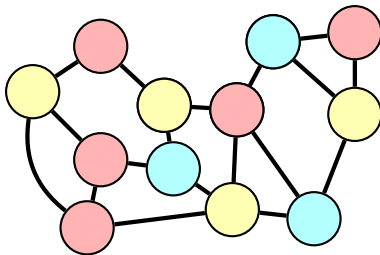
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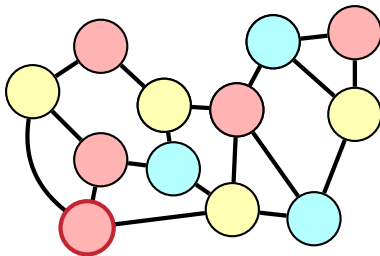
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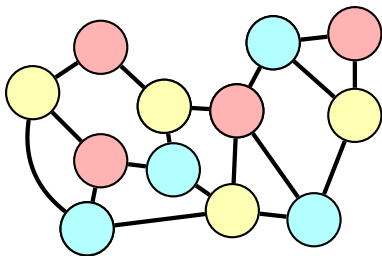
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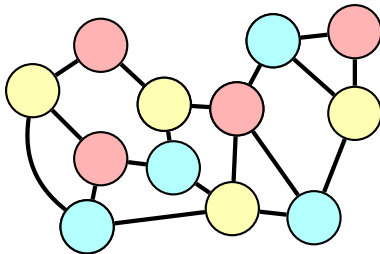
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The Petford–Welsh algorithm ...

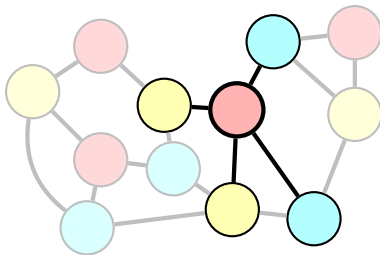
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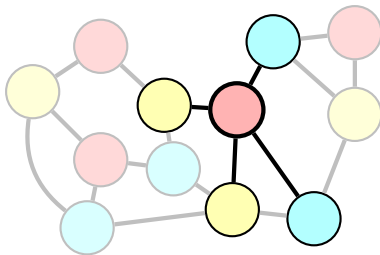


$$\omega^{-\mathcal{N}}(\text{red node}, \text{rainbow})$$

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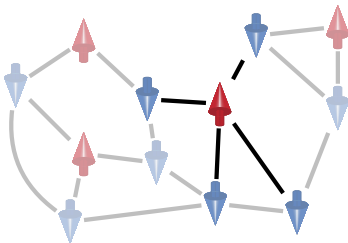


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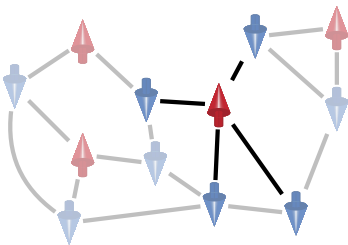


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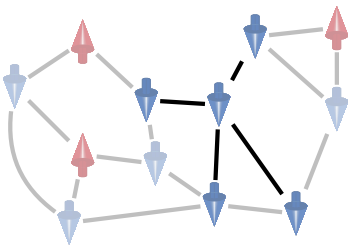
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$$\left(1 + e^{\Delta E / (k_B T)}\right)^{-1}$$

The Modified Petford–Welsh Algorithm

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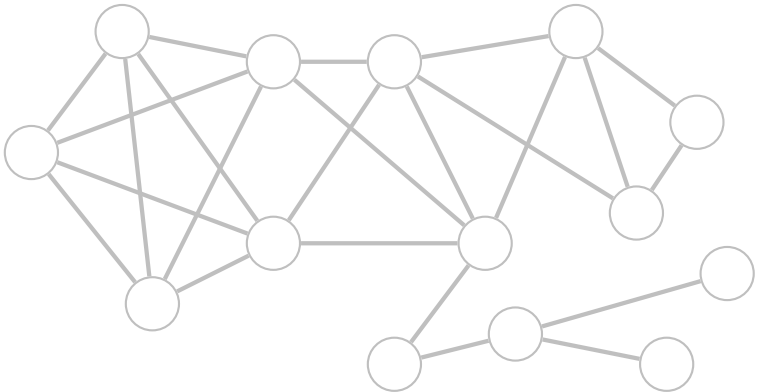
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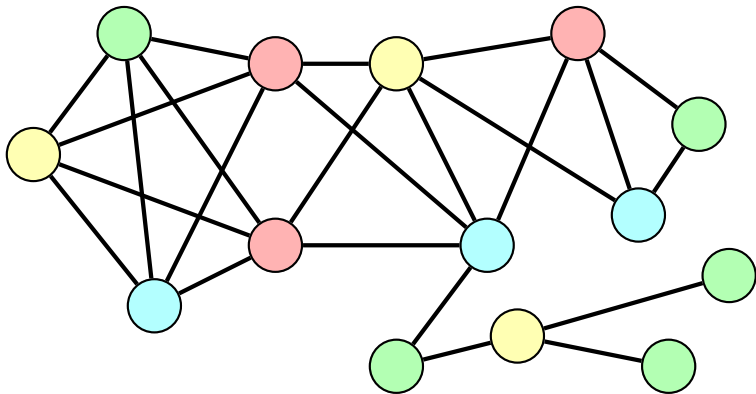
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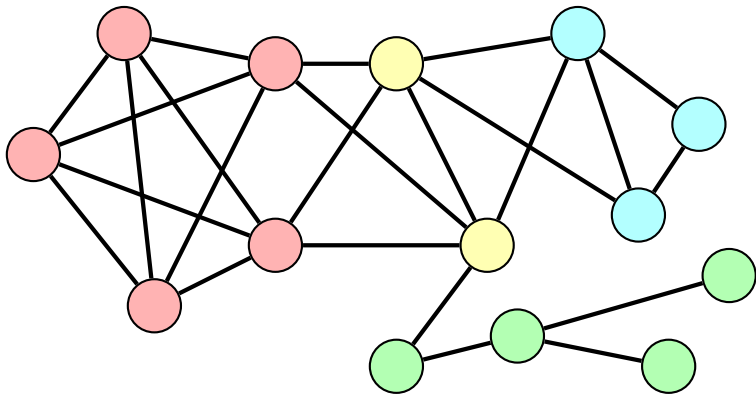


The Modified Petford–Welsh Algorithm

Graph colouring

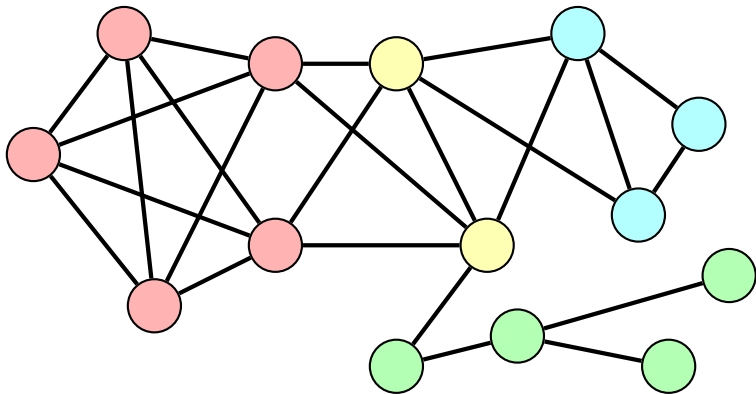


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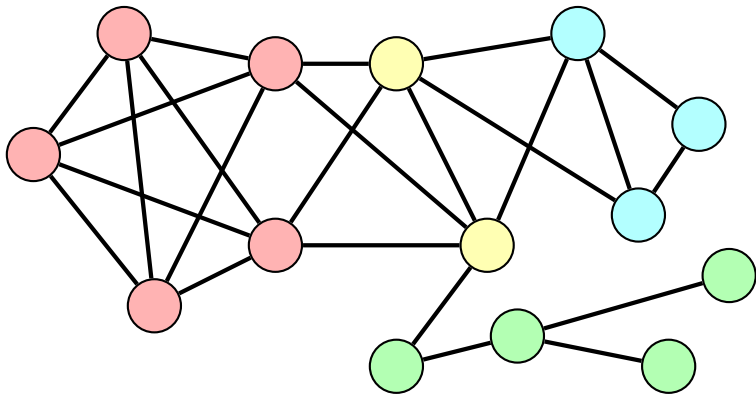
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Graph clustering



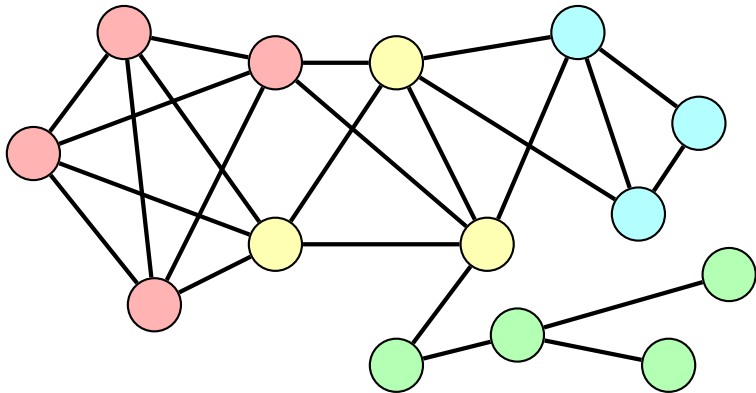
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clustering [partitioning or grouping data into *similar* subsets]



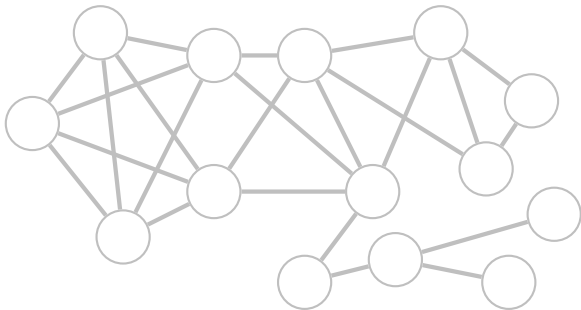
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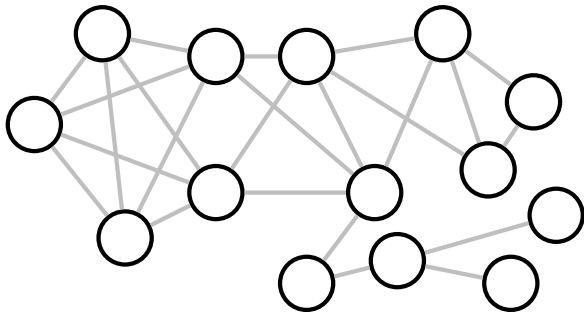
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The goal of **clustering** is to separate a given set of objects $X = \{x_1, x_2, \dots, x_n\}$ into non-overlapping groups/*clusters* $\mathcal{C} = \{C_1, C_2, \dots, C_m\}$ that satisfy



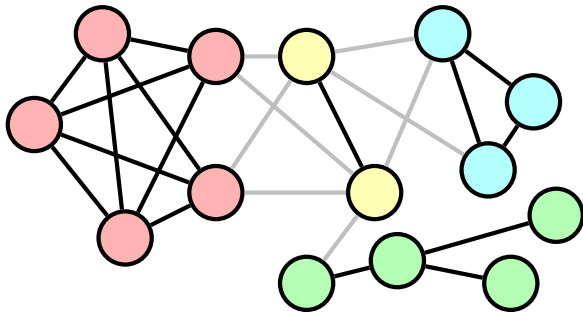
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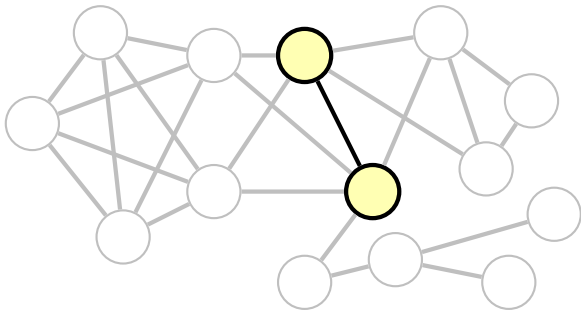
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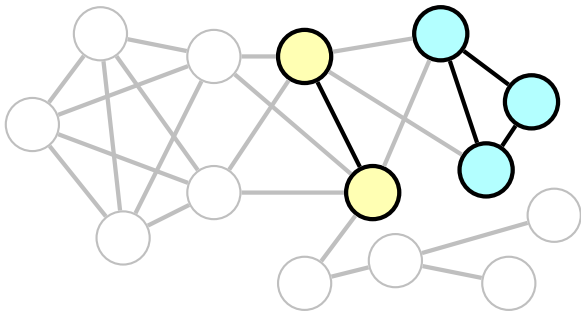


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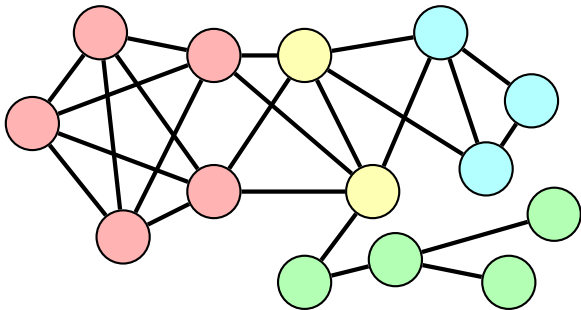
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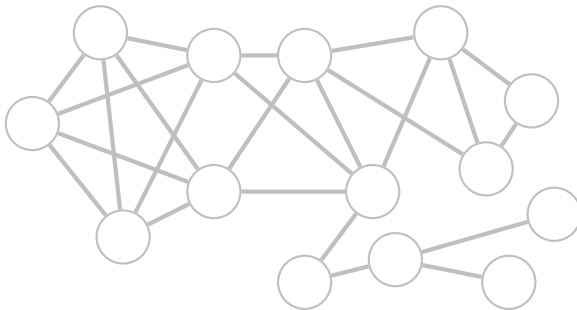
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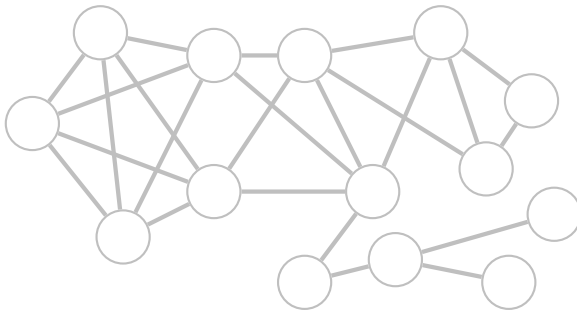
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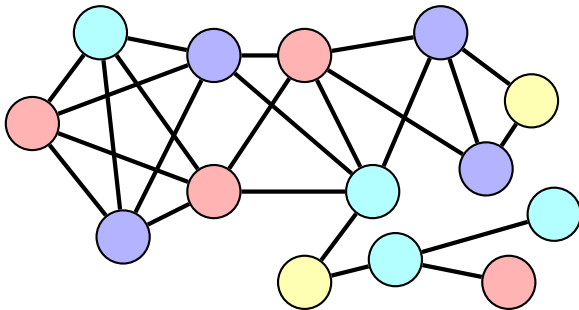
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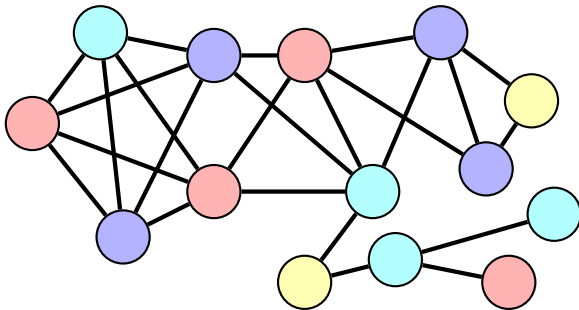
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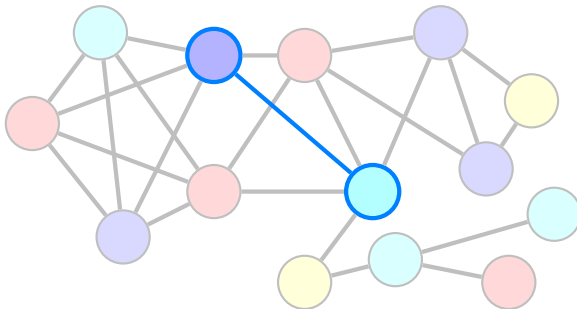
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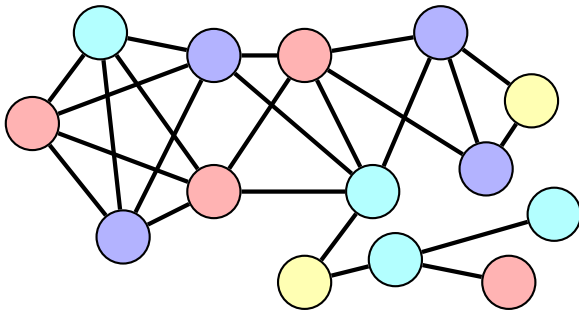
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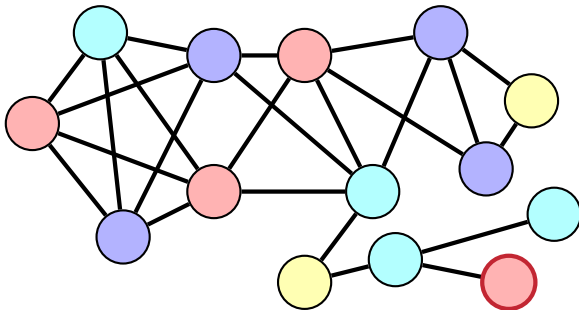
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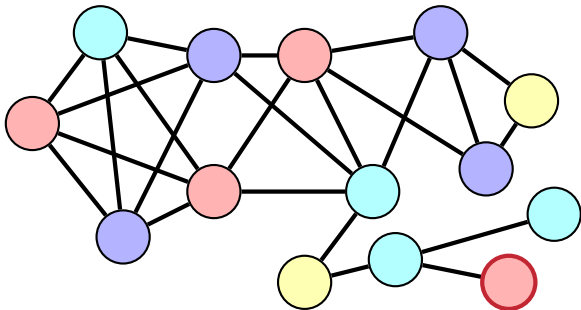
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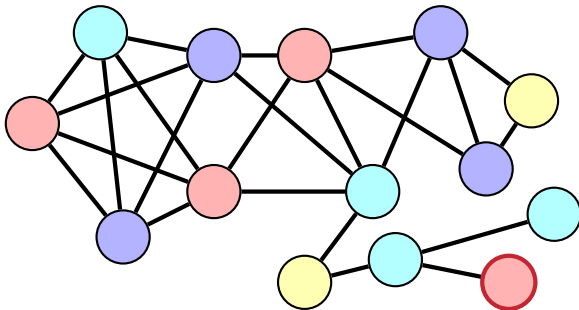
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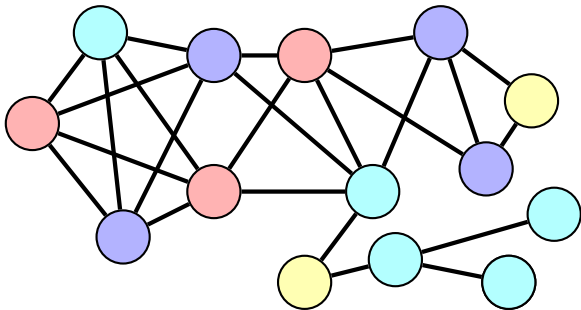
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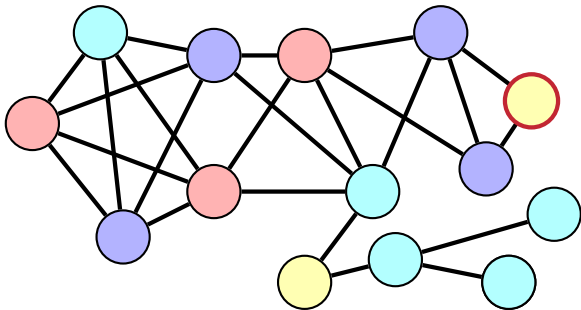
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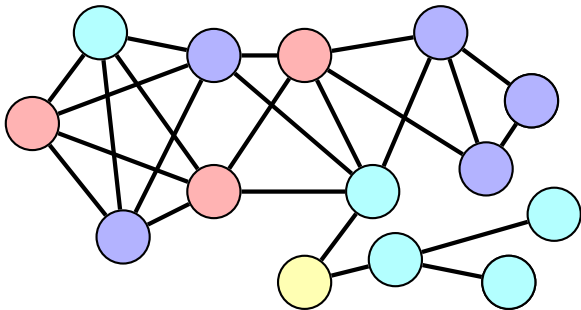
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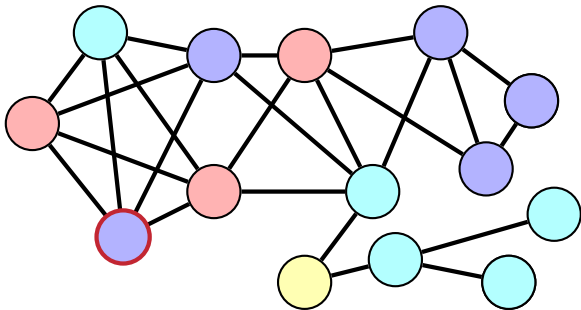
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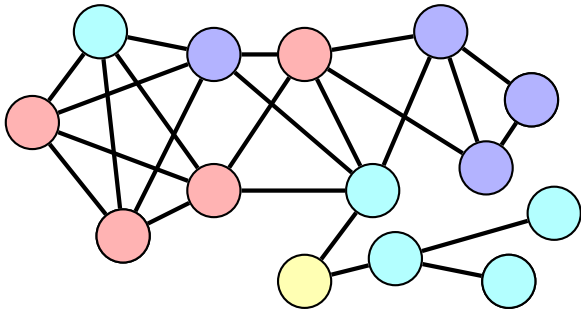
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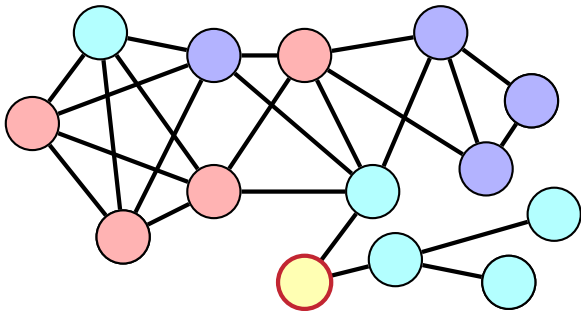
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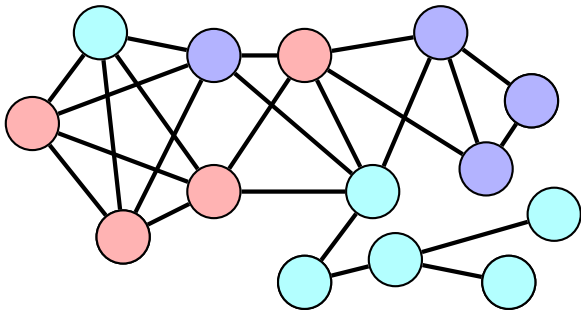
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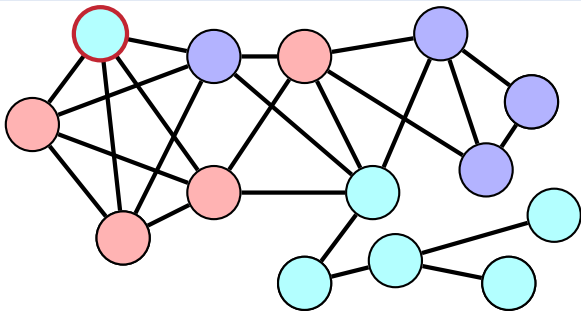
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A randomised *clustering* algorithm [<https://github.com/ikicab/mPW>]

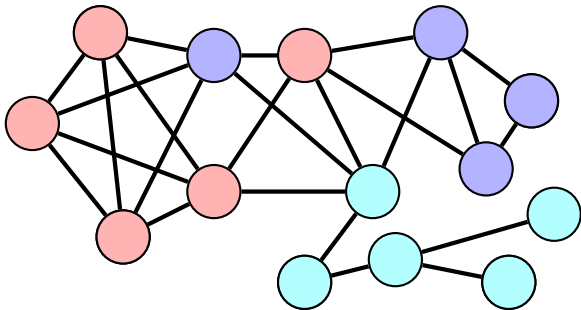
1. generate an initial *k*-clustering
2. **while** (there is a *bad vertex*) **and** ($\text{Var}[\textit{bad edges}] \geq \textit{tol}$) **repeat**
 - 2.1 choose a bad vertex v uniformly at random
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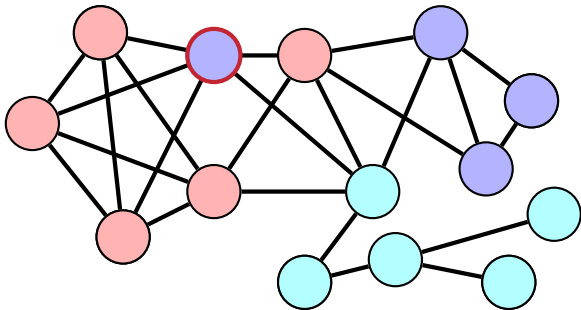
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The Modified Petford–Welsh Algorithm

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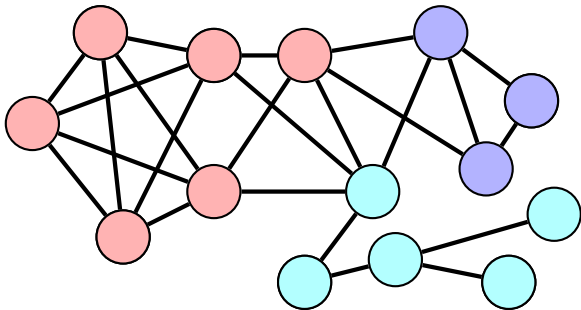
1. generate an initial *k*-clustering
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The Modified Petford–Welsh Algorithm

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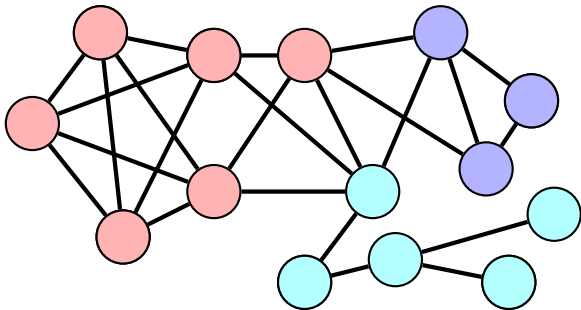
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The Modified Petford–Welsh Algorithm

Stopping Condition

$$\text{Var}_{\text{step}} = \text{Var}(\text{bad edges}[1 : \text{step}]) < \text{tol}$$

The Modified Petford–Welsh Algorithm

Stopping Condition

$$\text{Var}_{\text{step}} = \text{Var}(\text{bad edges}[\text{step} - L + 1 : \text{step}]) < \text{tol}$$

The Modified Petford–Welsh Algorithm

Stopping Condition

`bad edges = []`

$$\mu_L = \frac{1}{L}(b_1 + b_2 + \dots + b_L)$$

$$\text{Var}_L = \frac{1}{L-1}(b_1^2 + b_2^2 + \dots + b_L^2) - \frac{L}{L-1}\mu_L^2$$

The Modified Petford–Welsh Algorithm

Stopping Condition

step = 1 : **bad edges = $[b_1]$**

$$\mu_L = \frac{1}{L}(b_1 + b_2 + \dots + b_L)$$

$$\text{Var}_L = \frac{1}{L-1}(b_1^2 + b_2^2 + \dots + b_L^2) - \frac{L}{L-1}\mu_L^2$$

The Modified Petford–Welsh Algorithm

Stopping Condition

step = 1 : bad edges = $[b_1]$

$$\mu_L = \frac{1}{L}(b_1 + b_2 + \dots + b_L)$$

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The Modified Petford–Welsh Algorithm

Stopping Condition

step = 2 : bad edges = $[b_1, b_2]$

$$\mu_L = \frac{1}{L}(b_1 + b_2 + \dots + b_L)$$

$$\text{Var}_L = \frac{1}{L-1}(b_1^2 + b_2^2 + \dots + b_L^2) - \frac{L}{L-1}\mu_L^2$$

The Modified Petford–Welsh Algorithm

Stopping Condition

step = L : bad edges = $[b_1, b_2, \dots, b_L]$

$$\mu_L = \frac{1}{L}(b_1 + b_2 + \dots + b_L)$$

$$\text{Var}_L = \frac{1}{L-1}(b_1^2 + b_2^2 + \dots + b_L^2) - \frac{L}{L-1}\mu_L^2$$

The Modified Petford–Welsh Algorithm

Stopping Condition

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The Modified Petford–Welsh Algorithm

Stopping Condition

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The Modified Petford–Welsh Algorithm

Stopping Condition

step = $L + 1$: bad edges = $[b_1, b_2, \dots, b_L, b_{L+1}]$

$$\mu_L = \frac{1}{L}(b_1 + b_2 + \dots + b_L)$$

$$\text{Var}_L = \frac{1}{L-1}(b_1^2 + b_2^2 + \dots + b_L^2) - \frac{L}{L-1}\mu_L^2$$

The Modified Petford–Welsh Algorithm

Stopping Condition

step = $L + 1$: bad edges = $[b_1, b_2, \dots, b_L, b_{L+1}]$

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$$\mu_{L+1} = \mu_L + \frac{1}{L}(b_{L+1} - b_1)$$

The Modified Petford–Welsh Algorithm

Stopping Condition

step = $L + 1$: bad edges = $[b_1, b_2, \dots, b_L, b_{L+1}]$

$$\mu_L = \frac{1}{L}(b_1 + b_2 + \dots + b_L)$$

$$\text{Var}_L = \frac{1}{L-1}(b_1^2 + b_2^2 + \dots + b_L^2) - \frac{L}{L-1}\mu_L^2$$

$$\mu_{L+1} = \mu_L + \frac{1}{L}(b_{L+1} - b_1)$$

$$\text{Var}_{L+1} = \text{Var}_L + \frac{1}{L-1}(b_{L+1} - b_1)(b_{L+1} + b_1 - \mu_{L+1} - \mu_L)$$

The Modified Petford–Welsh Algorithm

Stopping Condition

step = $L + 2$: bad edges = $[b_1, b_2, \dots, b_L, b_{L+1}, b_{L+2}]$

$$\mu_L = \frac{1}{L}(b_1 + b_2 + \dots + b_L)$$

$$\text{Var}_L = \frac{1}{L-1}(b_1^2 + b_2^2 + \dots + b_L^2) - \frac{L}{L-1}\mu_L^2$$

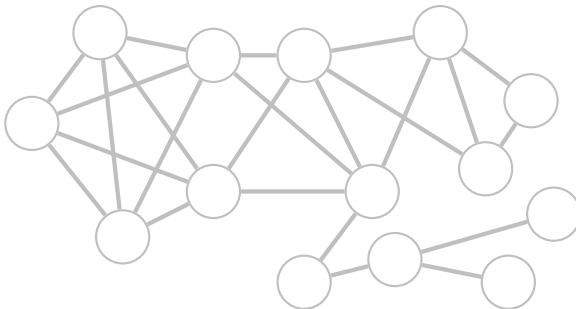
$$\mu_{L+2} = \mu_{L+1} + \frac{1}{L}(b_{L+2} - b_2)$$

$$\text{Var}_{L+2} = \text{Var}_{L+1} + \frac{1}{L-1}(b_{L+2} - b_2)(b_{L+2} + b_2 - \mu_{L+2} - \mu_{L+1})$$

The Modified Petford–Welsh Algorithm

Fine-tuning

Problems

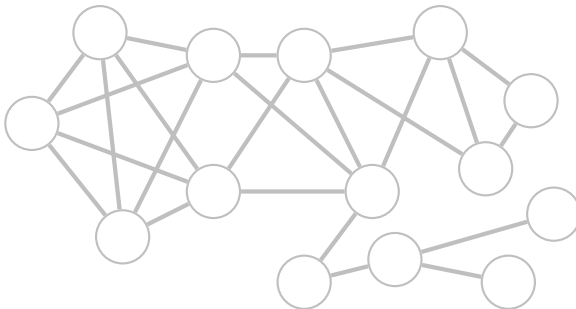


The Modified Petford–Welsh Algorithm

Fine-tuning

Problems

- different clusters get assigned the same colour due to random seeds

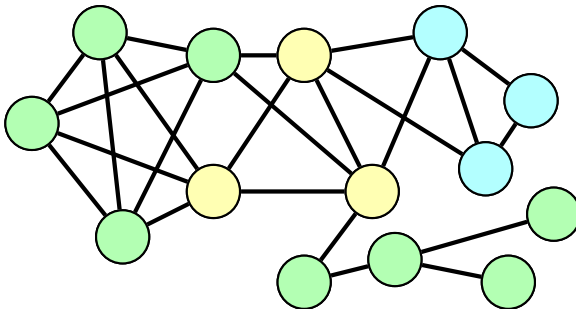


The Modified Petford–Welsh Algorithm

Fine-tuning

Problems

- different clusters get assigned the **same colour** due to random seeds

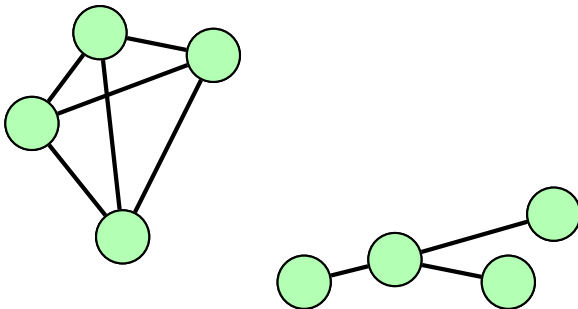


The Modified Petford–Welsh Algorithm

Fine-tuning

Problems

- different clusters get assigned the same colour due to random seeds

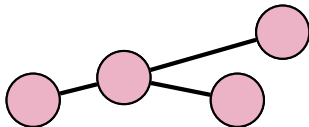
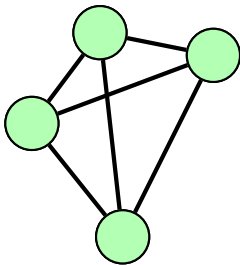


The Modified Petford–Welsh Algorithm

Fine-tuning

Problems

- different clusters get assigned the same colour due to random seeds

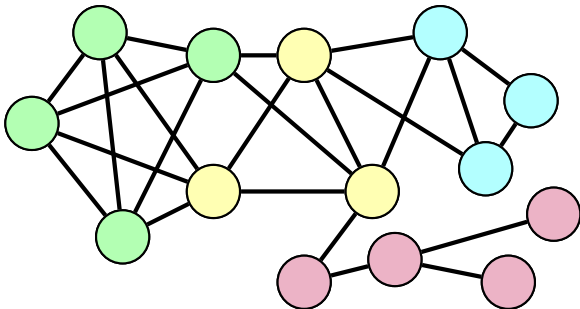


The Modified Petford–Welsh Algorithm

Fine-tuning

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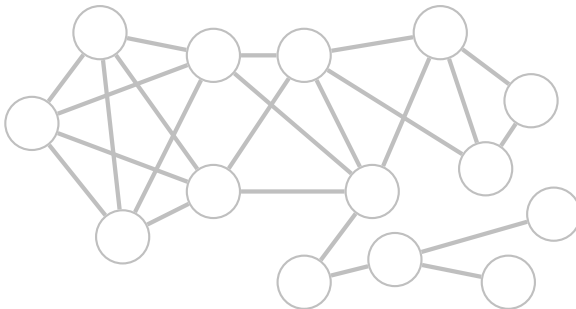


The Modified Petford–Welsh Algorithm

Fine-tuning

Problems

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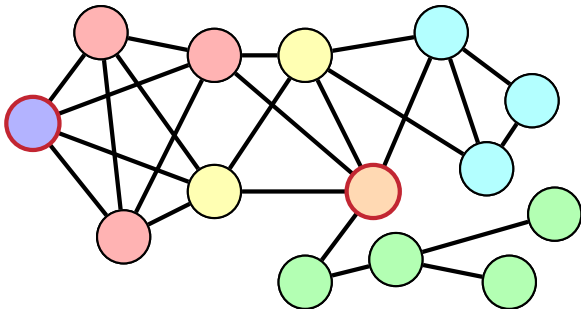


The Modified Petford–Welsh Algorithm

Fine-tuning

Problems

- different clusters get assigned the same colour due to random seeds
- **outliers** (singleton clusters)

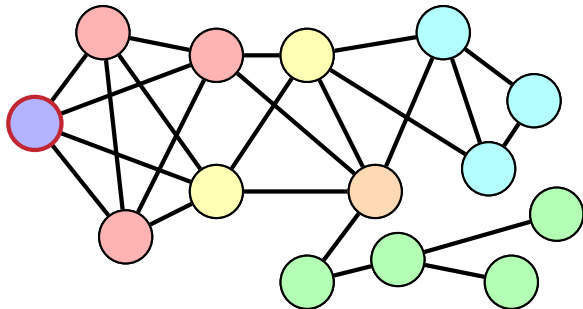


The Modified Petford–Welsh Algorithm

Fine-tuning

Problems

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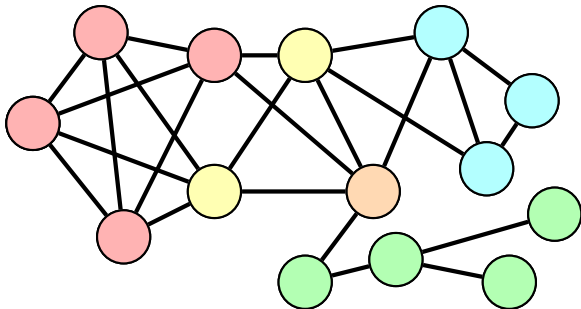


The Modified Petford–Welsh Algorithm

Fine-tuning

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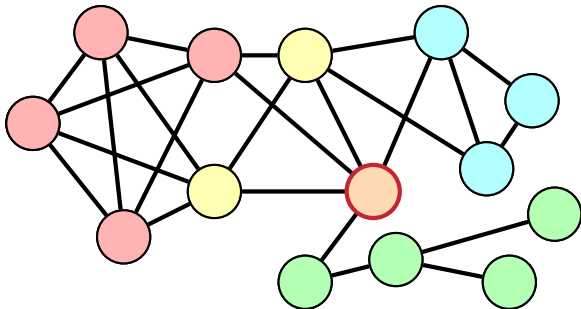


The Modified Petford–Welsh Algorithm

Fine-tuning

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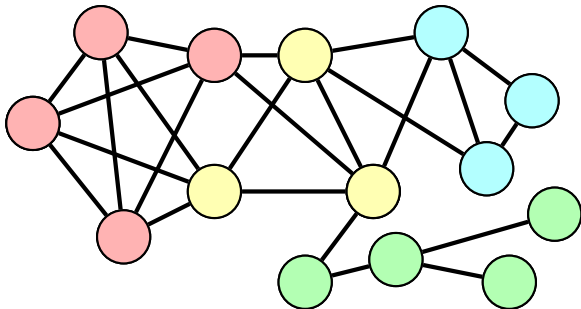


The Modified Petford–Welsh Algorithm

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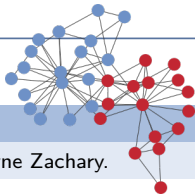


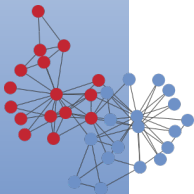
The Modified Petford–Welsh Algorithm

Experiments

Zachary ($|V| = 34, |E| = 78$)

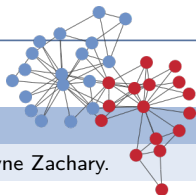
Ties among the members of a university karate club by Wayne Zachary.





The Modified Petford–Welsh Algorithm

Experiments



Zachary ($|V| = 34, |E| = 78$)

Ties among the members of a university karate club by Wayne Zachary.

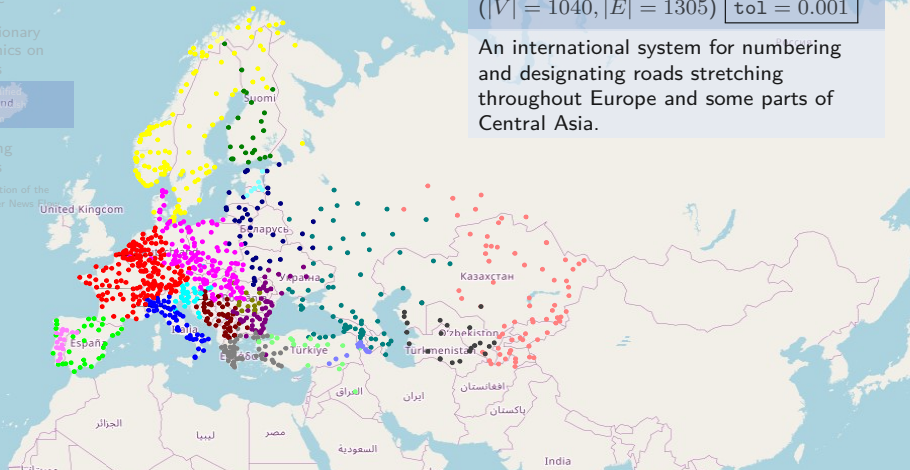
| Method | NMI | ARI | ϕ | γ | Q | $ C $ |
|---------------------|--------------|--------------|--------------|--------------|--------------|-------|
| Edge betweenness | 0.517 | 0.392 | 0.424 | 0.692 | 0.401 | 5 |
| Fastgreedy | 0.576 | 0.568 | 0.574 | 0.756 | 0.381 | 3 |
| Infomap | 0.578 | 0.591 | 0.668 | 0.821 | 0.402 | 3 |
| Label propagation | 0.865 | 0.882 | 0.773 | 0.949 | 0.415 | 3 |
| Leading eigenvector | 0.612 | 0.435 | 0.487 | 0.667 | 0.393 | 4 |
| Multilevel | 0.516 | 0.392 | 0.558 | 0.731 | 0.419 | 4 |
| Spinglass | 0.627 | 0.509 | 0.563 | 0.756 | 0.420 | 4 |
| Walktrap | 0.531 | 0.321 | 0.434 | 0.590 | 0.353 | 5 |
| mPW | 1.000 | 1.000 | 0.773 | 0.949 | 0.403 | 2 |

The Modified Petford–Welsh Algorithm

Experiments

International E-road network
($|V| = 1040, |E| = 1305$) $\tau_{01} = 0.001$

An international system for numbering and designating roads stretching throughout Europe and some parts of Central Asia.

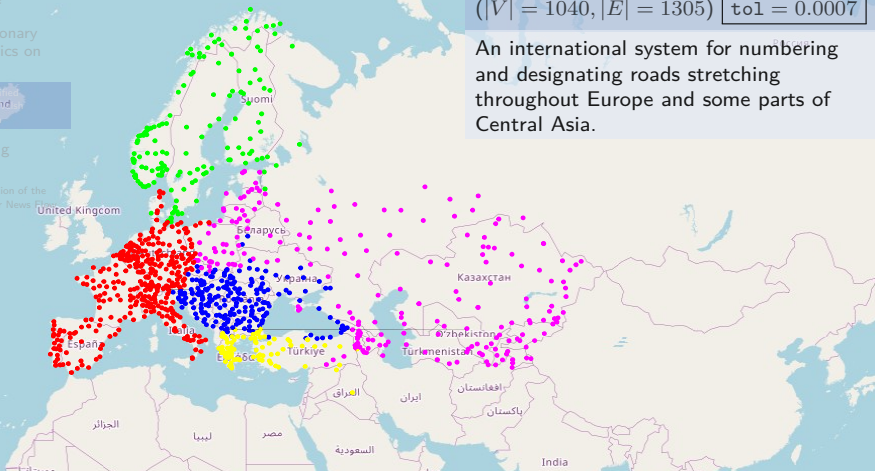


The Modified Petford–Welsh Algorithm

Experiments

International E-road network
($|V| = 1040, |E| = 1305$) $\tau_{01} = 0.0007$

An international system for numbering and designating roads stretching throughout Europe and some parts of Central Asia.



The Modified Petford–Welsh Algorithm

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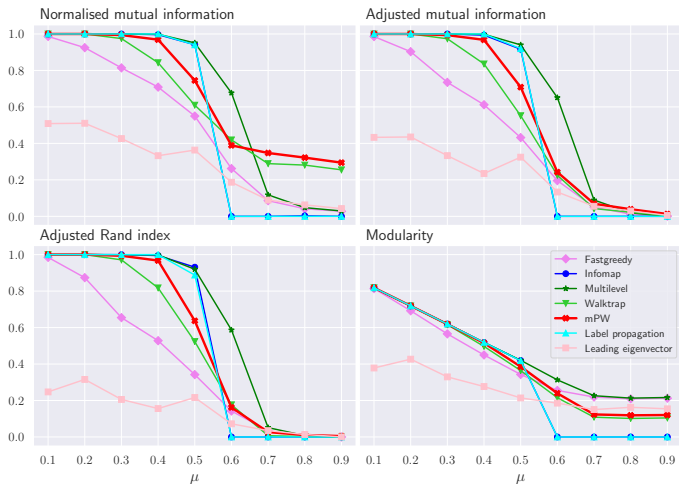
International E-road network
($|V| = 1040, |E| = 1305$)

An international system for numbering and designating roads stretching throughout Europe and some parts of Central Asia.

| Method | ϕ | γ | Q | $ C $ |
|---------------------|--------------|--------------|--------------|-------|
| Edge betweenness | – | – | – | – |
| Fastgreedy | 0.860 | 0.917 | 0.861 | 24 |
| Infomap | 0.663 | 0.787 | 0.777 | 126 |
| Label propagation | 0.731 | 0.856 | 0.828 | 82 |
| Leading eigenvector | 0.794 | 0.887 | 0.835 | 26 |
| Multilevel | 0.873 | 0.921 | 0.867 | 24 |
| Spinglass | 0.866 | 0.924 | 0.872 | 25 |
| Walktrap | 0.757 | 0.886 | 0.828 | 67 |
| mPW | 0.945 | 0.979 | 0.845 | 17 |

The Modified Petford–Welsh Algorithm

Experiments



LFR ($|V| = 1000, \gamma = 2, \beta = 1, k_{avg} = 15, k_{max} = 100, c_{min} = 50, c_{max} = 100$)

Co-evolution of the Multilayer News Flow

Science Contents News Careers Journals

SHARE REPORT

The spread of true and false news online

Soroush Vosoughi¹, Deb Roy¹, Sinan Aral^{2,*}

¹Massachusetts Institute of Technology (MIT), the Media Lab, E14-526, 75 Amherst Street, Cambridge, MA 02142, USA.

²MIT, E62-364, 100 Main Street, Cambridge, MA 02142, USA.

*Corresponding author. Email: sinan@mit.edu
—Hide authors and affiliations

Science 09 Mar 2018:
Vol. 359, Issue 6380, pp. 1146–1151
DOI: 10.1126/science.aap9559

Article Figures & Data Info & Metrics eLetters PDF

Lies spread faster than the truth

There is worldwide concern over false news and the possibility that it can influence political, economic, and social well-being. To understand how false news spreads, Vosoughi *et al.* used a data set of rumor cascades on Twitter from 2006 to 2017. About 126,000 rumors were spread by ~3 million people. False news reached more people than the truth; the top 1% of false news cascades diffused to between 1000 and 100,000 people, whereas the truth rarely diffused to more than 1000 people. Falsehood also diffused faster than the truth. The degree of novelty and the emotional reactions of recipients may be responsible for the differences observed.

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Science
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09 March 2018

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Vosoughi, S., Roy, D. & Aral, S. The spread of true and false news online, *Science* **359**(6380) (2018), 1146–1151.

Co-evolution of the Multilayer News Flow



news

any asserted claim

Co-evolution of the Multilayer News Flow



news

any asserted claim

rumour



the social phenomena of a news
diffusing through a network

Co-evolution of the Multilayer News Flow



news

any asserted claim

rumour



the social phenomena of a news
diffusing through a network



**rumour
cascade**

an uninterrupted chain of posts
stemming from a single common
assertion about a news story

Co-evolution of the Multilayer News Flow



news

any asserted claim

rumour



the social phenomena of a news
diffusing through a network



**rumour
cascade**

an uninterrupted chain of posts
stemming from a single common
assertion about a news story

**true/false
news**



a news story that can be
objectively verified/disproved

Co-evolution of the Multilayer News Flow

RESEARCH

SOCIAL SCIENCE

The spread of true and false news online

Soroush Vosoughi,¹ Deb Roy,¹ Sinan Aral^{2*}

We investigated the differential diffusion of all of the verified true and false news stories distributed on Twitter from 2006 to 2017. The data comprise ~126,000 stories tweeted by ~3 million people more than 4.5 million times. We classified news as true or false using information from six independent fact-checking organizations that exhibited 95 to 98% agreement on the classifications. Falsehood diffused significantly farther, faster, deeper, and more broadly than the truth in all categories of information, and the effects were more pronounced for false political news than for false news about terrorism, natural disasters, science, urban legends, or financial information. We found that false news was more novel than true news, which suggests that people were more likely to share novel information. Whereas false stories inspired fear, disgust, and surprise in replies, true stories inspired anticipation, sadness, joy, and trust. Contrary to conventional wisdom, robots accelerated the spread of true and false news at the same rate, implying that false news spreads more than the truth because humans, not robots, are more likely to spread it.

Foundational theories of decision-making (1–3), cooperation (4), communication (5), and markets (6) all view some conceptualization of truth or accuracy as central to the functioning of nearly every human endeavor. Yet, both true and false information

Current work analyzes the spread of single rumors, like the discovery of the Higgs boson (13) or the Haitian earthquake of 2010 (14), and multiple rumors from a single disaster event, like the Boston Marathon bombing of 2013 (10), or it develops theoretical models of rumor diffusion

support their positions as unreliable or fake news, whereas sources that support their positions are labeled reliable or not fake, the term has lost all connection to the actual veracity of the information presented, rendering it meaningless for use in academic classification. We have therefore explicitly avoided the term fake news throughout this paper and instead use the more objectively verifiable terms “true” or “false” news. Although the terms fake news and misinformation also imply a willful distortion of the truth, we do not make any claims about the intent of the purveyors of the information in our analyses. We instead focus our attention on veracity and stories that have been verified as true or false.

We also purposefully adopt a broad definition of the term news. Rather than defining what constitutes news on the basis of the institutional source of the assertions in a story, we refer to any asserted claim made on Twitter as news (we defend this decision in the supplementary materials section on “reliable sources,” section S1.2). We define news as any story or claim with an assertion in it and a rumor as the social phenomena of a news story or claim spreading or diffusing through the Twitter network. That is, rumors are inherently social and involve the sharing of claims between people. News, on the other hand, is an assertion with claims, whether it is shared or not.

A rumor cascade begins on Twitter when a

Co-evolution of the Multilayer News Flow

Falsehood diffused significantly **farther, faster, deeper,** and more **broadly** than the truth in **all** categories of information.

Co-evolution of the Multilayer News Flow

Falsehood diffused significantly farther, faster, deeper, and more broadly than the truth in all categories of information.

Whereas the truth rarely diffused to more than 1000 people, the top 1% of false-news cascades routinely diffused to between 1000 and 100,000 people. [...] It took the truth about **six times** as long as falsehood to reach 1500 people and **20 times** as long as falsehood to reach a cascade depth of 10. [...] falsehoods were **70% more likely** to be retweeted than the truth, even when controlling for [user characteristics]

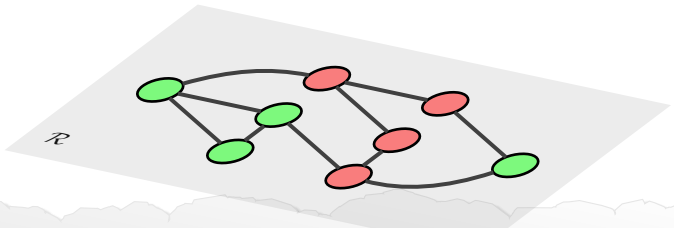
Co-evolution of the Multilayer News Flow

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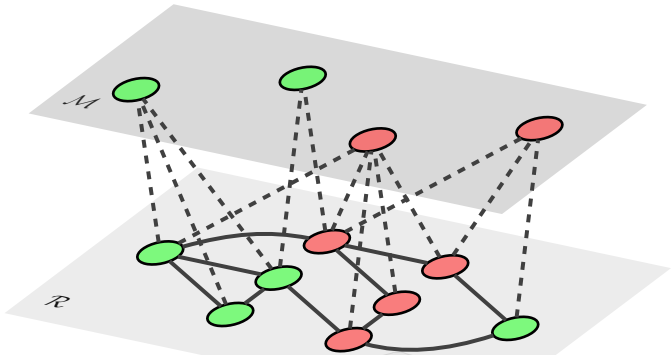
[...] falsehood did not simply spread through **broadcast dynamics** but rather through **peer-to-peer diffusion** characterized by a viral branching process.

Co-evolution of the Multilayer News Flow



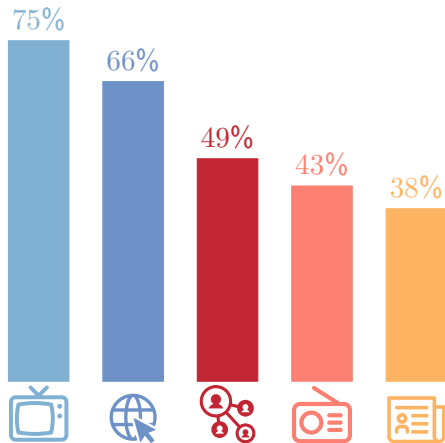
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Co-evolution of the Multilayer News Flow



[...] falsehood did not simply spread through **broadcast dynamics** but rather through **peer-to-peer diffusion** characterized by a viral branching process.

Co-evolution of the Multilayer News Flow



Co-evolution of the Multilayer News Flow

Motivation

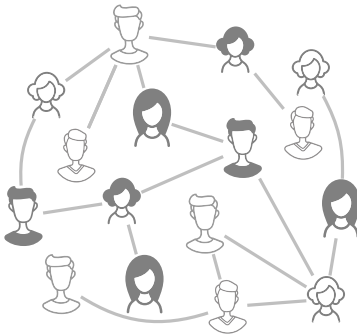
Outline

Evolutionary
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Graphs

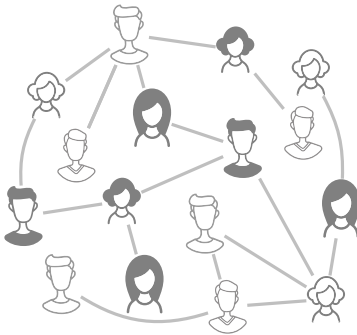
The Modified
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Evolving
Graphs

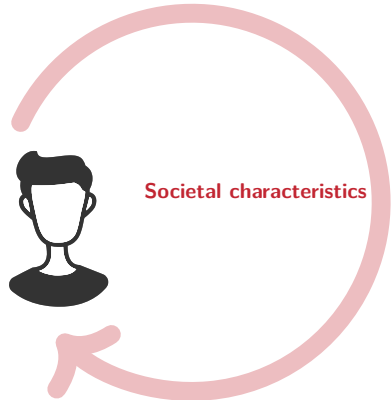
Co-evolution of the
Multilayer News Flow



Co-evolution of the Multilayer News Flow



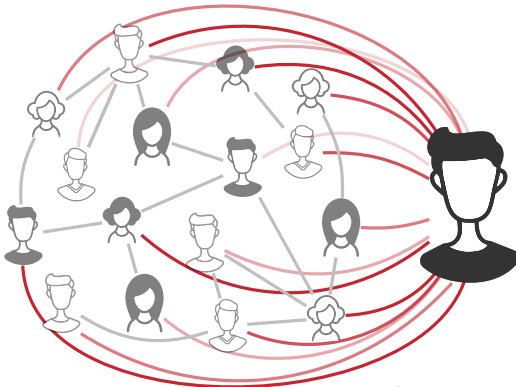
Human characteristics



Social media by design

Co-evolution of the Multilayer News Flow

Human characteristics



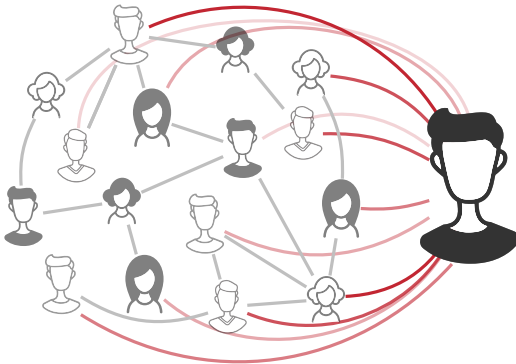
Societal characteristics

Social media by design

- fast-paced, massive influx of information

Co-evolution of the Multilayer News Flow

Human characteristics



Societal characteristics

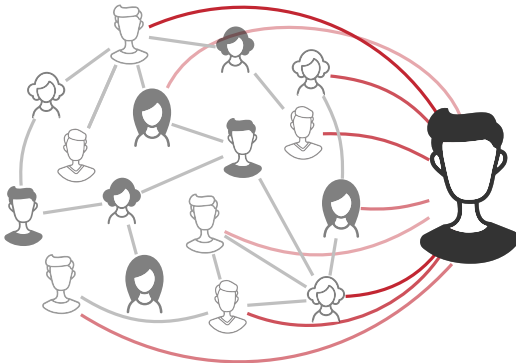
- Dunbar's number, homophily

Social media by design

- fast-paced, massive influx of information

Co-evolution of the Multilayer News Flow

Human characteristics



Societal characteristics

- Dunbar's number, homophily

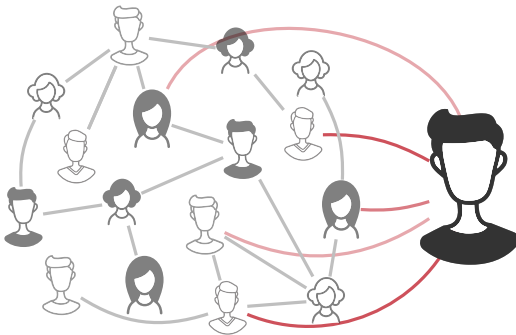
Social media by design

- fast-paced, massive influx of information
- engagement-boosting business model

Co-evolution of the Multilayer News Flow

Human characteristics

- limited time, short attention span



Societal characteristics

- Dunbar's number, homophily

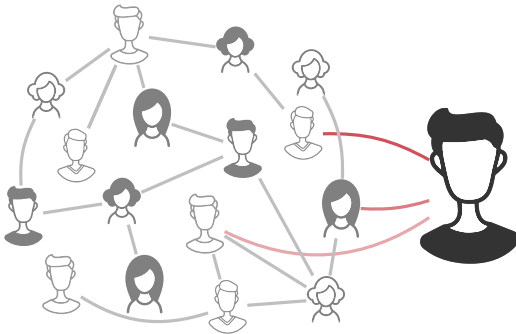
Social media by design

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Co-evolution of the Multilayer News Flow

Human characteristics

- limited time, short attention span
- confirmation bias, familiarity heuristics



Societal characteristics

- Dunbar's number, homophily

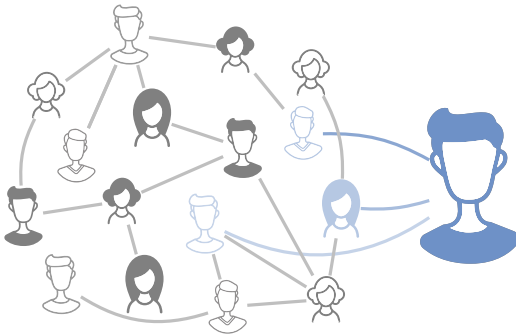
Social media by design

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Co-evolution of the Multilayer News Flow

Human characteristics

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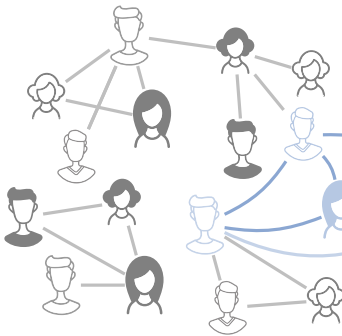
Societal characteristics

- Dunbar's number, homophily
- conformity bias

Social media by design

- fast-paced, massive influx of information
- engagement-boosting business model

Co-evolution of the Multilayer News Flow



Human characteristics

- limited time, short attention span
- confirmation bias, familiarity heuristics

Societal characteristics

- Dunbar's number, homophily
- conformity bias

Social media by design

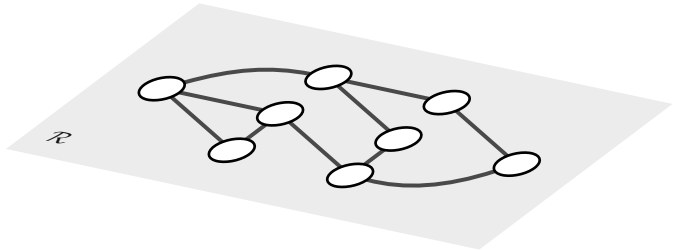
- fast-paced, massive influx of information
- engagement-boosting business model

**Filter bubbles &
echo chambers**



Co-evolution of the Multilayer News Flow

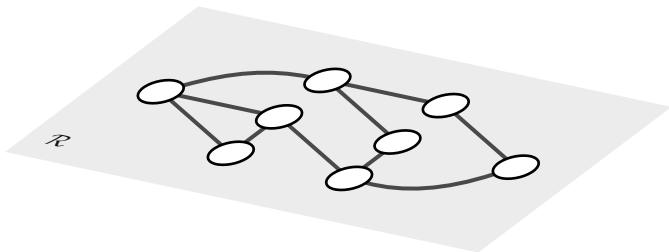
$$G_R = (\mathcal{R}, E_R)$$



Co-evolution of the Multilayer News Flow

$$G_R = (\mathcal{R}, E_R)$$

Barabási–Albert ($P(k) \sim k^{-\gamma}$, $\gamma = 2.5$)
 $\Delta(G_R) \lesssim$ Dunbar's number

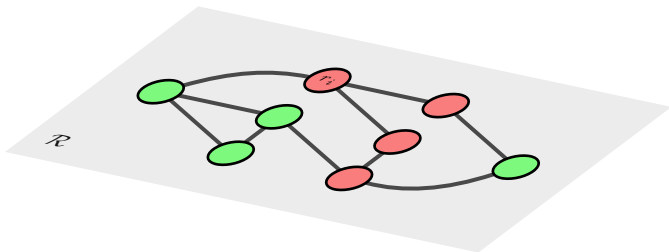


Co-evolution of the Multilayer News Flow

$$G_R = (\mathcal{R}, E_R)$$

$$r_i(t) \in \{0, 1\}$$

Barabási–Albert ($P(k) \sim k^{-\gamma}$, $\gamma = 2.5$)
 $\Delta(G_R) \lesssim$ Dunbar's number



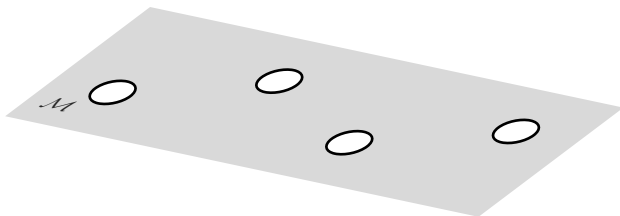
Co-evolution of the Multilayer News Flow

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Barabási–Albert ($P(k) \sim k^{-\gamma}$, $\gamma = 2.5$)
 $\Delta(G_R) \lesssim$ Dunbar's number

$$G_M = (\mathcal{M}, E_M)$$



Co-evolution of the Multilayer News Flow

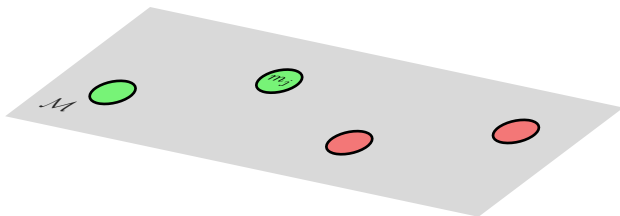
$$G_R = (\mathcal{R}, E_R)$$

$$r_i(t) \in \{0, 1\}$$

Barabási–Albert ($P(k) \sim k^{-\gamma}$, $\gamma = 2.5$)
 $\Delta(G_R) \lesssim$ Dunbar's number

$$G_M = (\mathcal{M}, E_M)$$

$$m_j \in \{0, 1\}$$



Co-evolution of the Multilayer News Flow

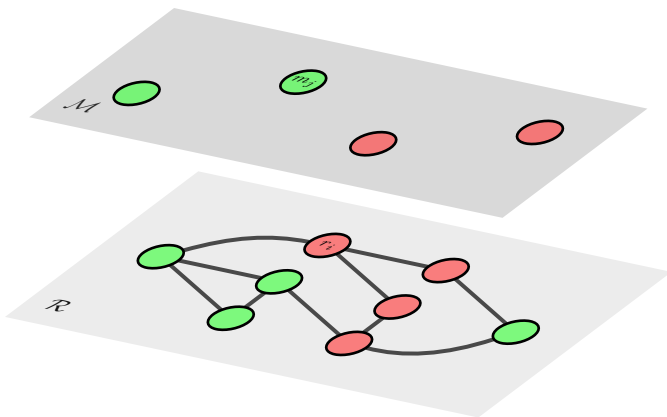
$$G_R = (\mathcal{R}, E_R)$$

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$$G_M = (\mathcal{M}, E_M)$$

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Co-evolution of the Multilayer News Flow

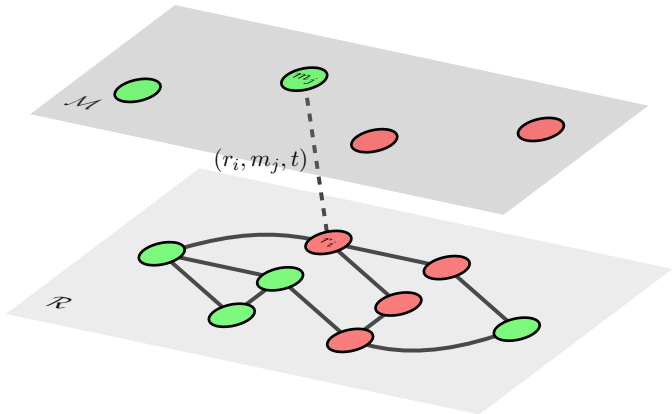
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Co-evolution of the Multilayer News Flow

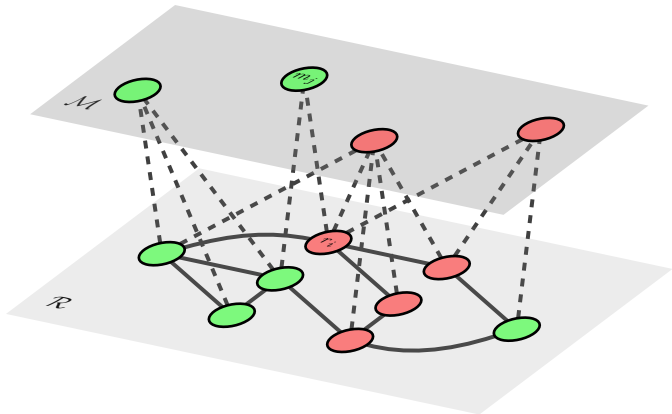
$$G_R = (\mathcal{R}, E_R)$$

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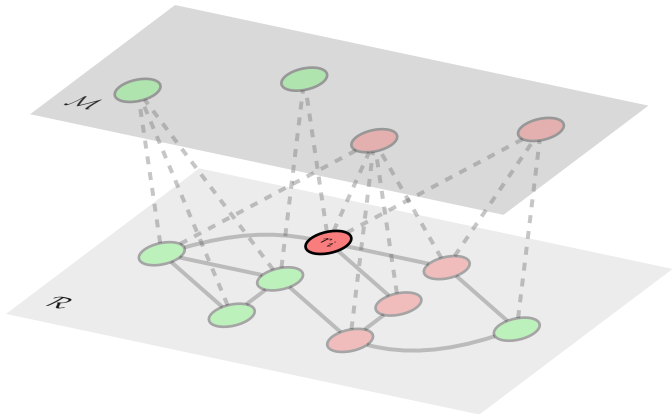
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Barabási-Albert ($P(k) \sim k^{-\gamma}$, $\gamma = 2.5$)
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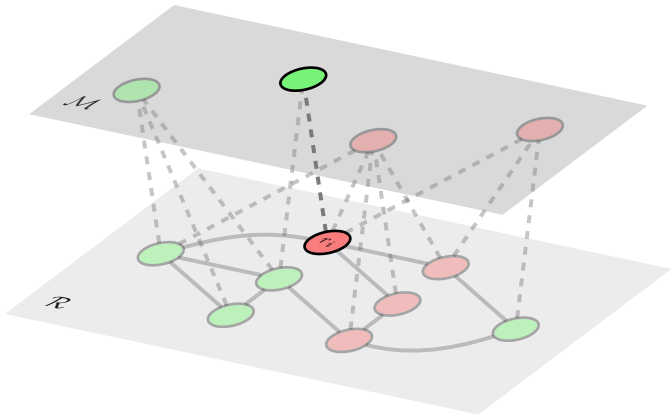
Co-evolution of the Multilayer News Flow

Media repertoire $(r_i^T(t), r_i^F(t))$ $r_i^T(t) + r_i^F(t) \leq M \ll R$



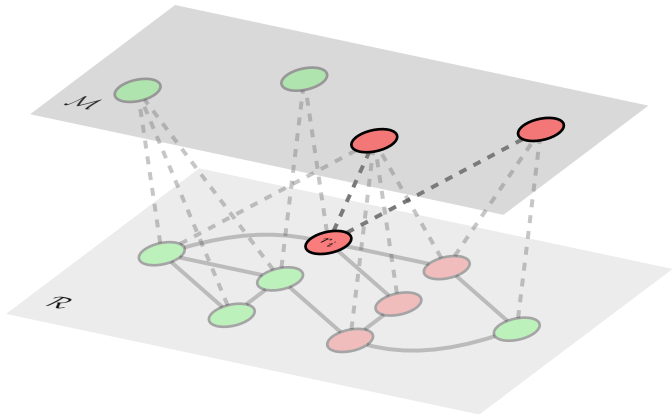
Co-evolution of the Multilayer News Flow

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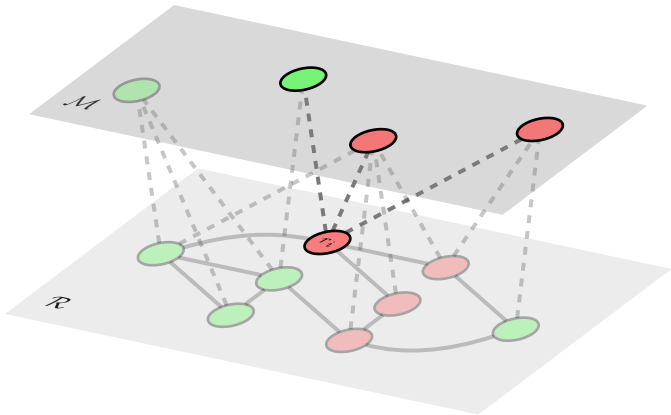
Co-evolution of the Multilayer News Flow

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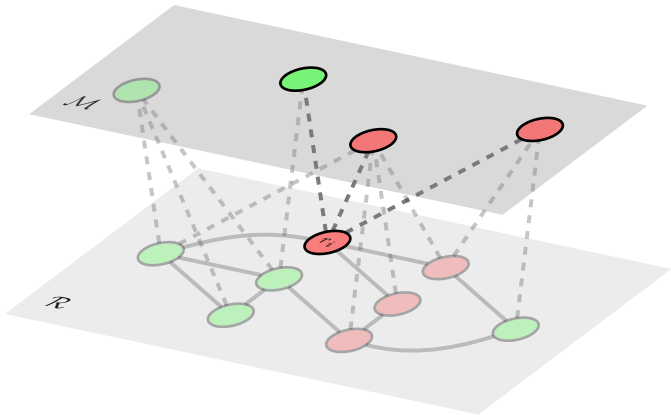
Co-evolution of the Multilayer News Flow

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Co-evolution of the Multilayer News Flow

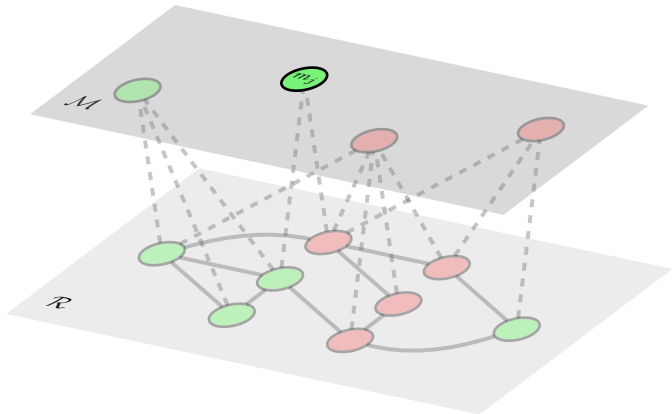
Media repertoire $(\rho_i^T(t), \rho_i^F(t))$ $r_i^T(t) + r_i^F(t) \leq M \ll R$



Co-evolution of the Multilayer News Flow

Media repertoire $(r_i^T(t), r_i^F(t))$ $r_i^T(t) + r_i^F(t) \leq M \ll R$

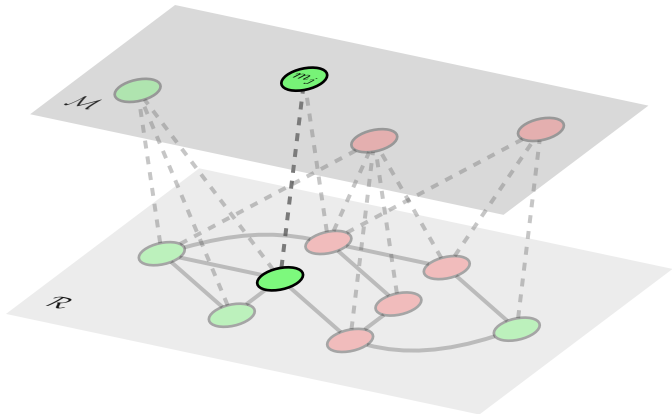
Media clientele $(m_j^T(t), m_j^F(t))$



Co-evolution of the Multilayer News Flow

Media repertoire $(r_i^T(t), r_i^F(t))$ $r_i^T(t) + r_i^F(t) \leq M \ll R$

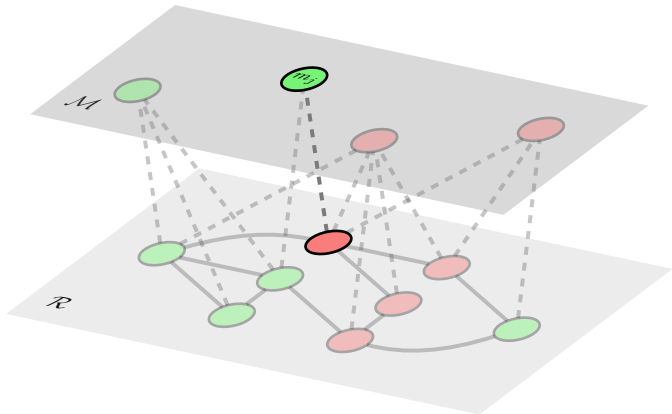
Media clientele $(m_j^T(t), m_j^F(t))$



Co-evolution of the Multilayer News Flow

Media repertoire $(r_i^T(t), r_i^F(t))$ $r_i^T(t) + r_i^F(t) \leq M \ll R$

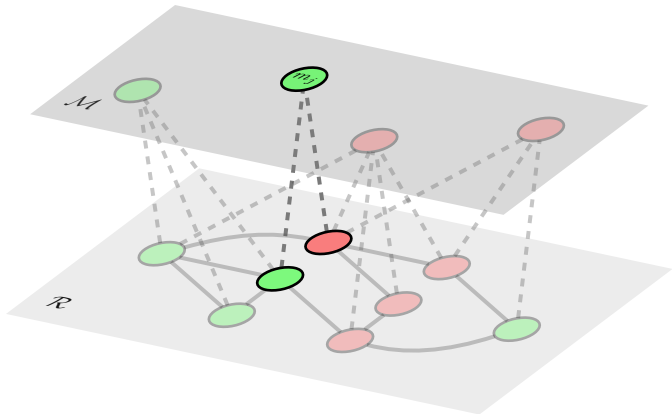
Media clientele $(m_j^T(t), m_j^F(t))$



Co-evolution of the Multilayer News Flow

Media repertoire $(r_i^T(t), r_i^F(t))$ $r_i^T(t) + r_i^F(t) \leq M \ll R$

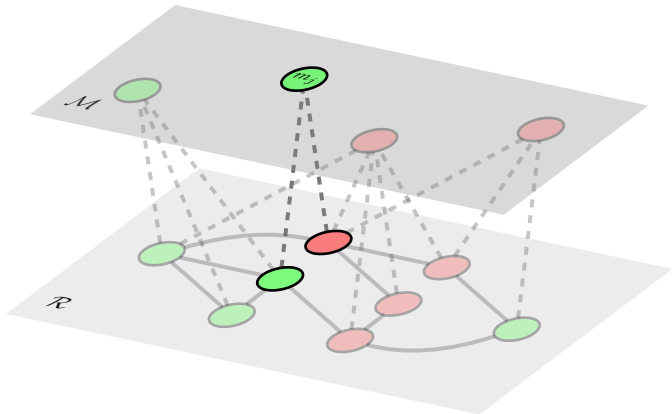
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Co-evolution of the Multilayer News Flow

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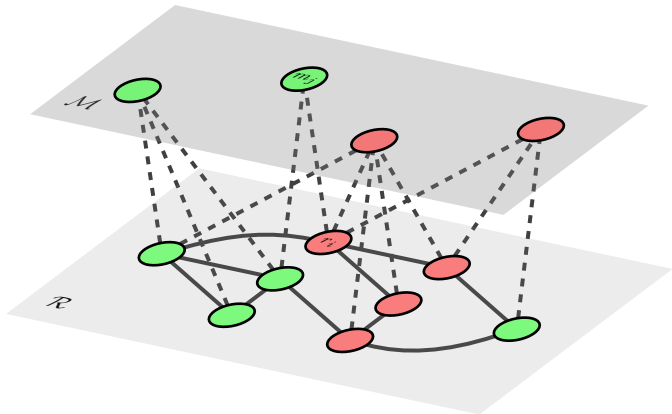
Media clientele $(\mu_j^T(t), \mu_j^F(t))$



Co-evolution of the Multilayer News Flow

Media repertoire $(r_i^T(t), r_i^F(t))$ $r_i^T(t) + r_i^F(t) \leq M \ll R$

Media clientele $(m_j^T(t), m_j^F(t))$

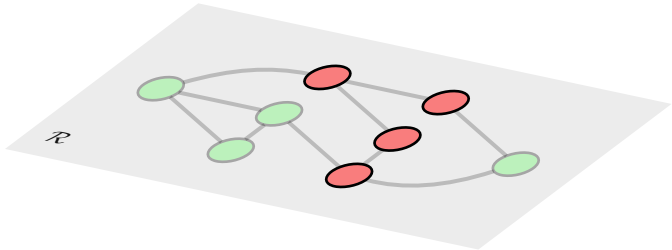


Co-evolution of the Multilayer News Flow

False-news followers

$$\mathcal{R}^F(t) = \{r_i \mid r_i(t) = 0\}$$

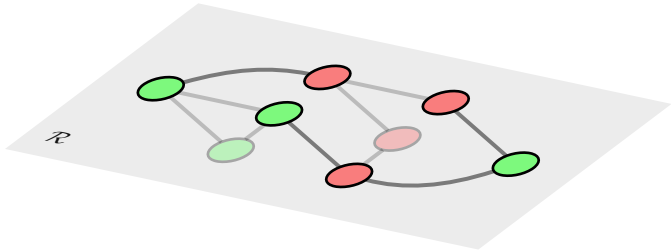
$$\rho^F(t)$$



Co-evolution of the Multilayer News Flow

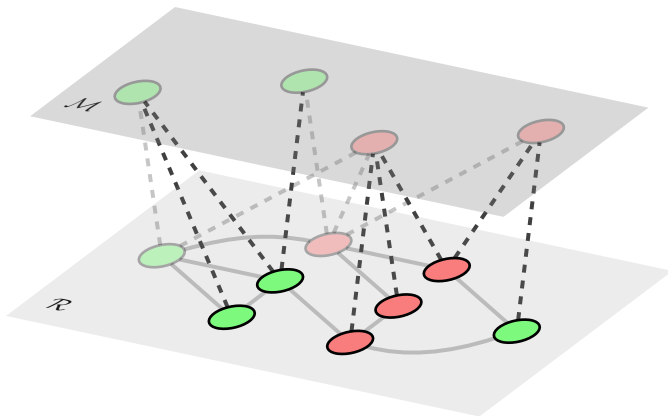
False-news followers $\mathcal{R}^F(t) = \{r_i \mid r_i(t) = 0\}$ $\rho^F(t)$

Unbalanced edges $E_R^{FT}(t) = \{r_i r_j \mid r_i(t) \neq r_j(t)\}$ $\rho^{FT}(t)$

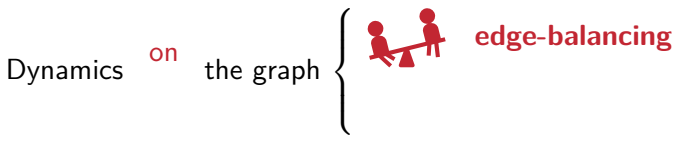


Co-evolution of the Multilayer News Flow

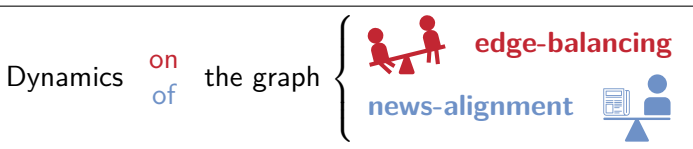
| | | |
|-----------------------|---|----------------|
| False-news followers | $\mathcal{R}^F(t) = \{r_i \mid r_i(t) = 0\}$ | $\rho^F(t)$ |
| Unbalanced edges | $E_R^{FT}(t) = \{r_i r_j \mid r_i(t) \neq r_j(t)\}$ | $\rho^{FT}(t)$ |
| Polarised individuals | $\mathcal{P}(t) = \{r_i \mid r_i^T(t) \cdot r_i^F(t) = 0\}$ | $\rho^P(t)$ |



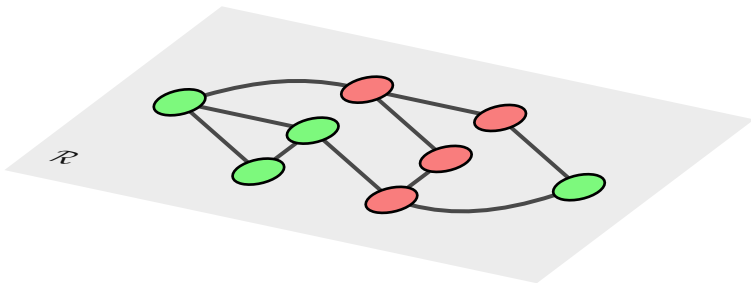
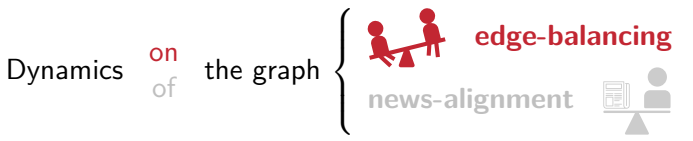
Co-evolution of the Multilayer News Flow



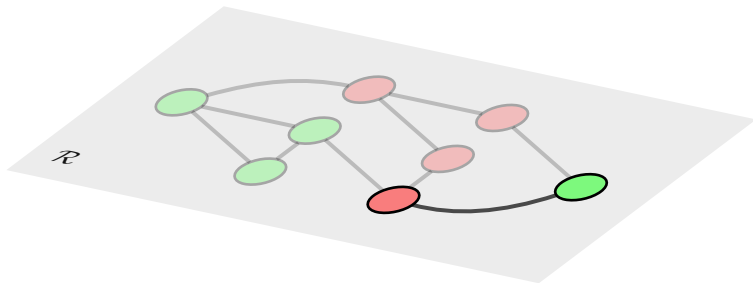
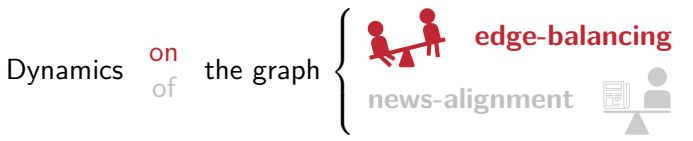
Co-evolution of the Multilayer News Flow



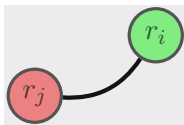
Co-evolution of the Multilayer News Flow

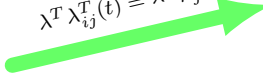


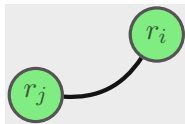
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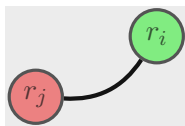
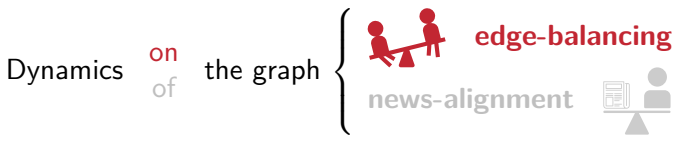
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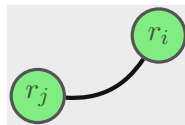
$$\lambda^T \lambda_{ij}^T(t) = \lambda^T \rho_j^T(t)$$




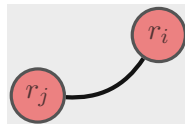
Co-evolution of the Multilayer News Flow



$$\lambda^T \lambda_{ij}^T(t) = \lambda^T \rho_j^T(t)$$



$$\lambda^F \lambda_{ij}^F(t) = \lambda^F \rho_i^F(t)$$



Co-evolution of the Multilayer News Flow

Dynamics

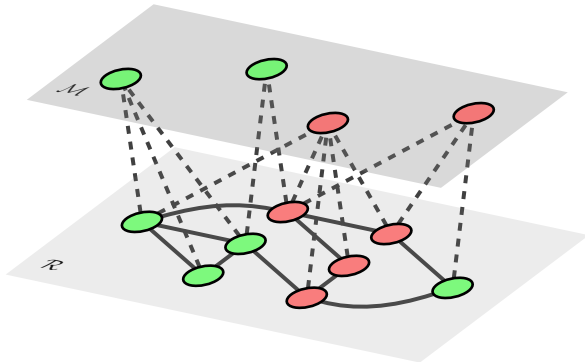
on
of

the graph

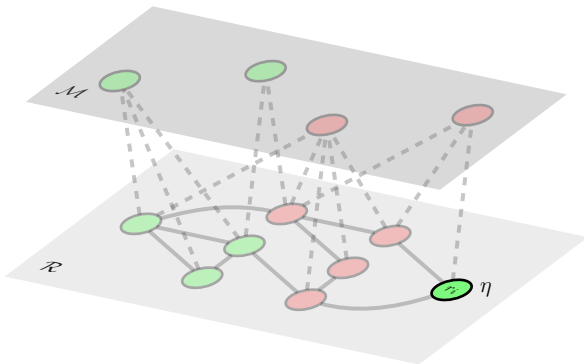
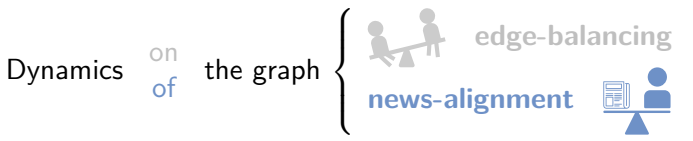


edge-balancing

news-alignment



Co-evolution of the Multilayer News Flow



Co-evolution of the Multilayer News Flow

Dynamics

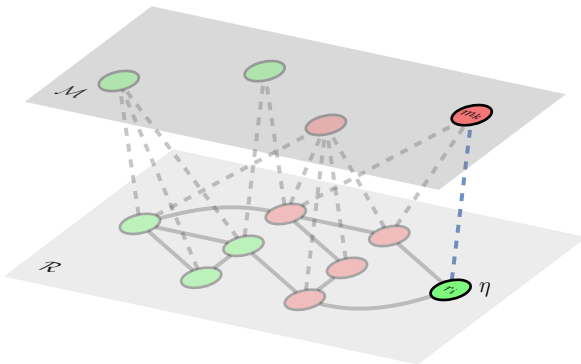
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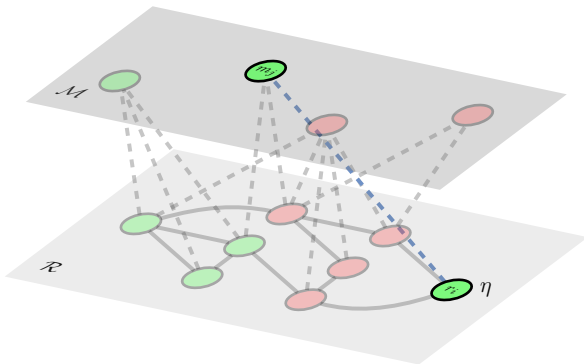
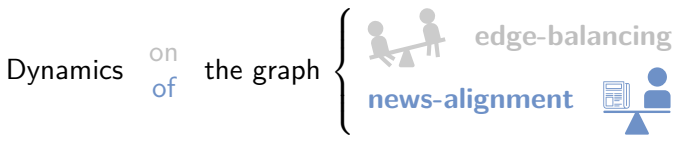


edge-balancing

news-alignment



Co-evolution of the Multilayer News Flow



Co-evolution of the Multilayer News Flow

Dynamics

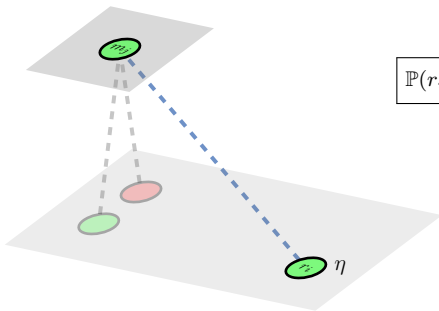
on
of

the graph



edge-balancing

news-alignment



$$\mathbb{P}(r_i \rightarrow m_j, t) \propto b_j(t) p_j^T(t)$$

Co-evolution of the Multilayer News Flow

Dynamics

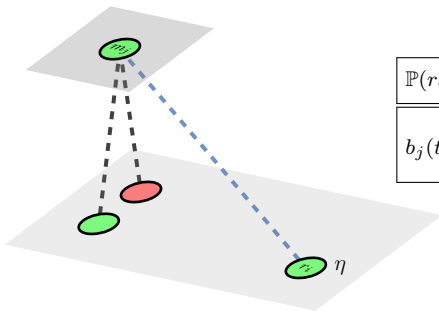
on
of

the graph



edge-balancing

news-alignment



$$\mathbb{P}(r_i \rightarrow m_j, t) \propto b_j(t) p_j^T(t)$$

$$b_j(t) := \frac{-\sum_S \mu_j^S(t) \log \mu_j^S(t)}{\log 2}$$

Co-evolution of the Multilayer News Flow

Dynamics

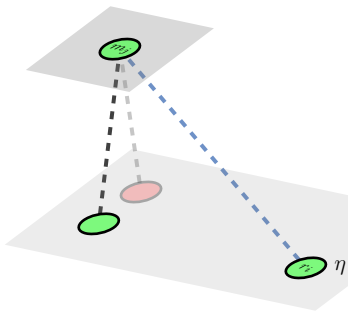
on
of

the graph



edge-balancing

news-alignment



$$\mathbb{P}(r_i \rightarrow m_j, t) \propto b_j(t) p_j^T(t)$$

$$b_j(t) := \frac{-\sum_S \mu_j^S(t) \log \mu_j^S(t)}{\log 2}$$

$$p_j^T(t) := \frac{1}{1 + e^{-m_j^T(t)}}$$

Co-evolution of the Multilayer News Flow

[<https://github.com/ikicab/NewsFlow>]

Motivation

Outline

Evolutionary
Dynamics on
Graphs

The Modified
Petford-Welsh
Algorithm

Evolving
Graphs

Co-evolution of the
Multilayer News Flow

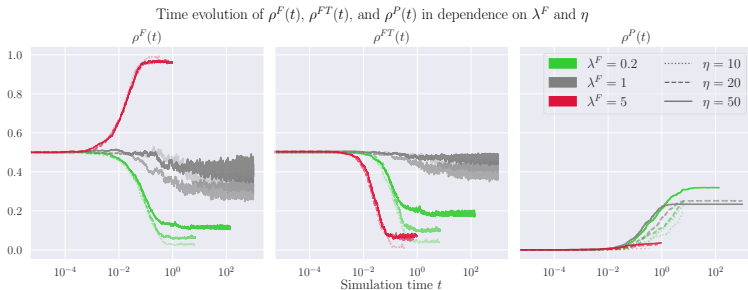
Gillespie algorithm

while $(\text{Var}_n(P(t_n)) \geq \text{tol})$ **and** $(\text{Var}_n(|E_R^{FT}(t_n)|) \geq \text{tol})$ **repeat**

1. $\lambda(t_n) = R\eta + \lambda^T \Lambda^T(t_n) + \lambda^F \Lambda^F(t_n)$
2. $\tau(t_n) = -\ln r / \lambda(t_n) = 1 / \lambda(t_n)$
3. **draw** $\mathcal{A} \in \{\mathcal{A}_N, \mathcal{A}_B^T, \mathcal{A}_B^F\}$ **at random:**
$$\mathbb{P}(\mathcal{A}_N) = \frac{R\eta}{\lambda(t_n)}, \quad \mathbb{P}(\mathcal{A}_B^T) = \frac{\lambda^T \Lambda^T(t_n)}{\lambda(t_n)}, \quad \mathbb{P}(\mathcal{A}_B^F) = \frac{\lambda^F \Lambda^F(t_n)}{\lambda(t_n)}$$
4. **draw either** $r_i \in \mathcal{R}$ **or** $r_i r_j \in E_R^{FT}(t_n)$ **at random:**
$$\mathbb{P}(r_i | \mathcal{A}_N) = \frac{1}{R}, \quad \mathbb{P}(r_i r_j | \mathcal{A}_B^T) = \frac{\lambda_{ij}^T(t_n)}{\Lambda^T(t_n)}, \quad \mathbb{P}(r_i r_j | \mathcal{A}_B^F) = \frac{\lambda_{ij}^F(t_n)}{\Lambda^F(t_n)}$$
5. **update the system;** $t_n \leftarrow t_n + \tau(t_n)$

Co-evolution of the Multilayer News Flow

Experiments [macro-scale dynamics]



Co-evolution of the Multilayer News Flow

Experiments [macro-scale dynamics]

Motivation

Outline

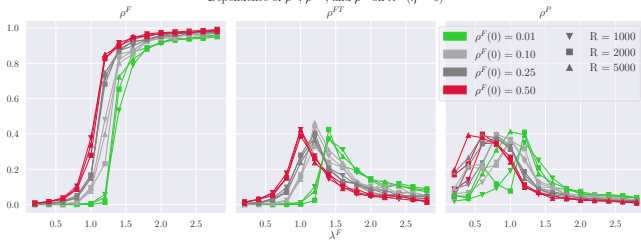
Evolutionary
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Petford-Welsh
Algorithm

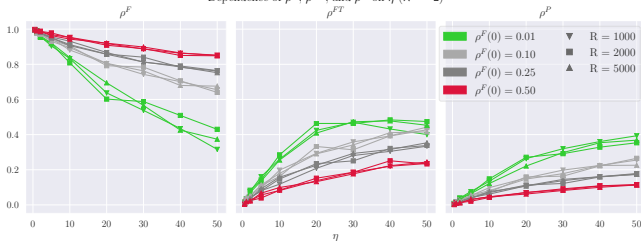
Evolving
Graphs

Co-evolution of the
Multilayer News Flow

Dependence of ρ^F , ρ^{FT} , and ρ^P on λ^F ($\eta = 5$)

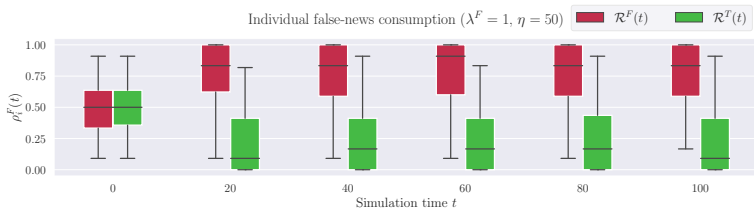
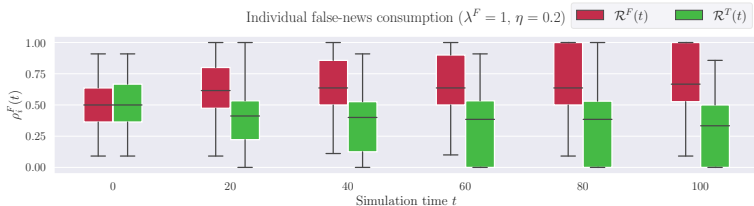


Dependence of ρ^F , ρ^{FT} , and ρ^P on η ($\lambda^F = 2$)



Co-evolution of the Multilayer News Flow

Experiments [micro-scale dynamics; news consumers]



Co-evolution of the Multilayer News Flow

Experiments [micro-scale dynamics; news consumers]

Motivation

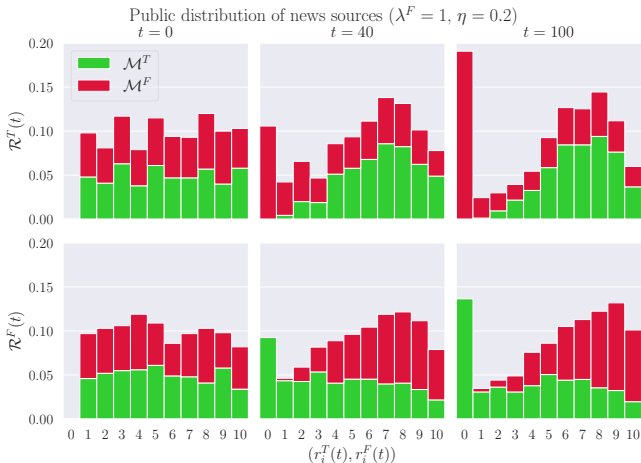
Outline

Evolutionary
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The Modified
Petford-Welsh
Algorithm

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Graphs

Co-evolution of the
Multilayer News Flow



Co-evolution of the Multilayer News Flow

Experiments [micro-scale dynamics; news consumers]

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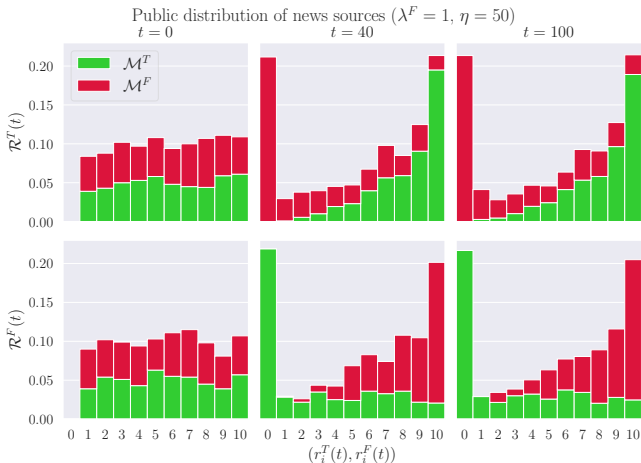
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Co-evolution of the Multilayer News Flow

Experiments [micro-scale dynamics; news providers]

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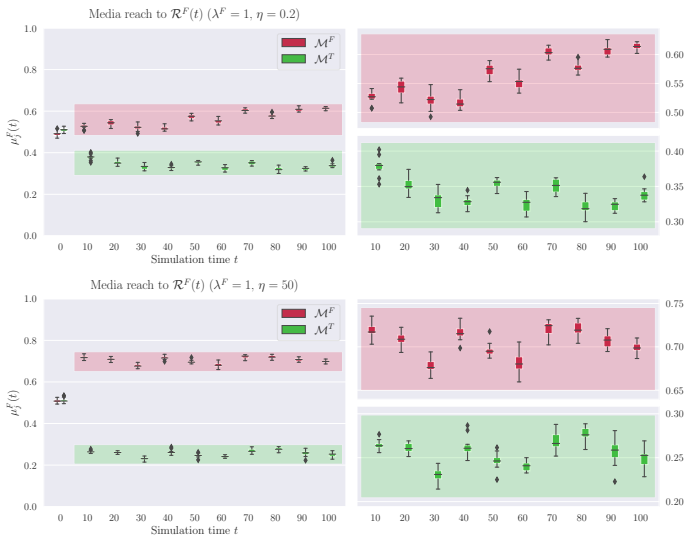
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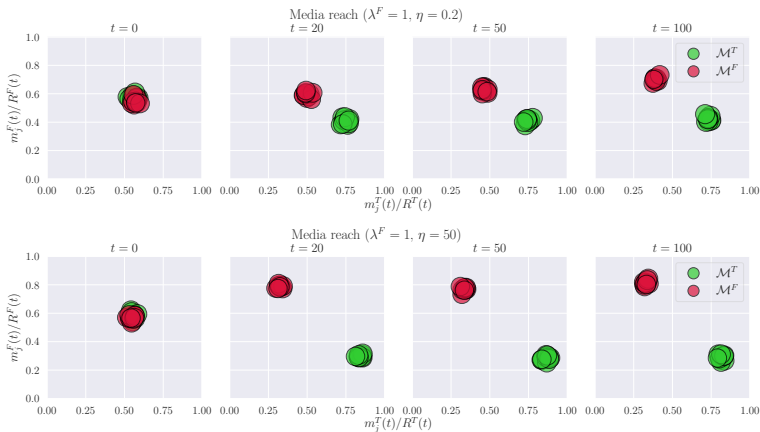
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Co-evolution of the Multilayer News Flow

Experiments [micro-scale dynamics; news providers]



*Thank
you*

